MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Arthur Neville Brown, M.A., Ludgrove, New Barnet, Herts;
Harry Cooper, 19 Cromer Road, Eastville, Bristol;
Phanindralal Gangooly, M.A., University, Calcutta, India; and
William Newsam McClean, 42 Durdham Park, Bristol,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Major-General Henry Herbert Lee (late R.E.), The Mount, Dinas Powis, near Cardiff, South Wales (proposed by T. E. Heath);

Augustus Edward Hough Love, D.Sc., F.R.S., Sedleian Professor of Natural Philosophy, Oxford (proposed by H. H. Turner);

Rev. Reginald Wm. Bickerton Moore, M.A. Oxon., Vicar of St James', Bath, 11 Devonshire Buildings, Bath (proposed by Rev. D. Higham Sparling); and

Herbert Gerard Tompkins, Examiner of Local Fund Accounts, N.W.P. and Oudh, Lahore, India (proposed by S. A. Saunder).

Anders Donner, Director of the Observatory, Helsingfors, Russia, was proposed by the Council as an Associate of the Society.

Eighty-eight presents were announced as having been received since the last meeting, including, amongst others :—

Besançon Observatory, Bulletin Astronomique, Bulletin Chronométrique, Bulletin Météorologique, etc., presented by the Observatory.

Astrographic Chart of the heavens ; 20 charts presented by the Royal Observatory, Greenwich ; 47 charts from the Paris, Algiers, and Toulouse Observatories, presented by the French Minister of Public Instruction.

Determinations of Personal Equation depending on Magnitude, made with the Transit Circles and the Heliometer at the Royal Observatory, Cape of Good Hope. By Sir David Gill, K.C.B., F.R.S., and S. S. Hough, F.R.S., H.M. Astronomer at the Cape.

The observations here discussed were planned at the time when a discussion had arisen between Mr Hinks and Dr Cohn * as to the existence of personality depending on magnitude in transit observations made by the Repsold travelling wire method.

The question at issue is so important in connection with the future of meridian observation, that it appeared desirable to devote a considerable amount of time and labour to its settlement. The position of a bright star relative to two symmetrically situated faint stars of equal magnitude can be determined by means of heliometer observations with great precision and absolute freedom from personality. With the modern heliometer the images of the brighter star can be reduced by means of screens to similarity with the images of the fainter stars. Even if we grant that there may be a personality depending on a residual apparent difference of magnitude between the image of the star whose brightness is reduced and that of the other star under measurement, it is impossible to imagine that the observer can be affected by any difference of personality in the measurement of distance of two pairs of stars where one component of one pair precedes and the other follows the star whose magnitude has been reduced by the screen, because the two conditions of measurement are exactly similar.

Farther, if the measures of both distances are made simultaneously (*i.e.* in the order a, b, b, a), the instantaneous scale-value must be the same for both. Then, if we suppose the position-angles also to be measured, we can introduce both the scale-value and the index error of the position-circle as unknown quantities, and determine the R.A. and Dec. of the brighter star relative to the fainter stars—free from all personality. And if the differences of declination are either very small or accurately determined, we

* *Monthly Notices*, lxvi. p. 481, and *Ast. Nach.*, No. 4119.

determine directly the R.A. of the bright star relative to the mean R.A. of the faint stars, free from personal and other systematic error.

We have in our experience at the Cape many instances of this freedom from personality in heliometer observations, of which a very good instance will be found in the *Annals of the Cape Observatory*, vol. viii. part 2, pp. 73B–81B, in connection with the determination of the parallax of β Crucis by Gill and Finlay. There we have observations of the distances of two stars of $6\frac{1}{2}$ and 7 magnitude in almost exactly opposite directions from β Crucis, and situated at distances of 2812″ and 3434″ respectively from it. Gill measured these distances on seventeen nights, and Finlay on fifteen nights. In forming the equations of condition for determining the parallax, the difference of the two distances for 1890 was assumed to be $622''{\cdot}050 + x_g$ for Gill, and $622''{\cdot}050 + x_f$ for Finlay. The simultaneous solution of the equations gave

$$x_g = -0''{\cdot}001 \pm 0''{\cdot}019$$
$$x_f = -0\ {\cdot}021 \pm 0\ {\cdot}023$$

and the probable error of the single observation $\pm 0''{\cdot}077$.

But if we regarded $x_g = x_f$, the probable error of the single observation became $\pm 0''{\cdot}074$.

The latter solution is thus the better of the two, proving that there is no sensible systematic difference in the determination of the position of β Crucis by the two observers.

About the time that the after-mentioned operations were commenced, the completion of the current Catalogue with the old Transit Circle left the observers free to make simultaneous observations with the latter, and a verification of their personal equation depending on magnitude, derived some two years previously by the screen method, was thus rendered as convenient as it was desirable.

The groups of stars selected consisted of three stars approximately on the arc of a great circle and, as nearly as could be chosen, equi-spaced. The middle star was on the average of about $3\frac{1}{2}$ magnitude, and the preceding and following stars on the average about $8\frac{1}{2}$ magnitude. The difference of R.A. was always large enough to allow of all three stars being observed without undue haste on the same night, while the distances were restricted by the limitation that they must fall within the range which could be measured by the heliometer, viz. about 2°.

The transit observations involved the transit of each of the three stars in succession, so that their differences of R.A. could be derived independently of clock error or other instrumental adjustments.

The heliometer measurements consisted in the measurements of distance and position-angle of the bright star from each of the fainter ones of the group. From these observations the position of the bright star relatively to the two fainter ones can be derived

independently of the instantaneous scale-value or index correction to the position-circle or other equatorial adjustments of the heliometer.

The adopted magnitudes of the stars have been taken where possible from the Harvard Photometry. For stars not contained therein, the magnitudes are derived from estimates made by the Transit Circle observers, using, as standards, stars whose magnitudes have been determined at Harvard.

The following is a list of the groups observed, numbered consecutively in order of R.A. The initials *a*, *b* are used to distinguish the preceding and following pairs of the group. It should be noticed that in some cases the same bright star has been used in more than one group. For simplicity of reduction, however, such groups have been treated as if they were independent.

TABLE I.

List of Magnitude Personal Equation Stars.

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0. h m s	Dec. ° ′ ″
1 *a*	−3°, 3882	8·7	16 4 41·01	− 3 17 56·17
	δ Ophiuchi	3·1	16 9 25·10	− 3 27 9·56
b	−3°, 3915	8·2	16 12 48·75	− 3 48 14·44
2 *a*	−25°, 5787	8·7	16 18 2·23	−26 3 4·54
	α Scorpii	1·3	16 23 38·51	−26 13 25·74
b	−26°, 5678	8·5	16 29 28·55	−26 16 58·14
3 *a*	−10°, 4383	8·3	16 40 10·37	−10 29 26·91
	20 Ophiuchi	4·7	16 44 37·97	−10 37 1·73
b	−10°, 4403	8·3	16 48 22·13	−10 35 56·57
4 *a*	−15°, 4439	8·7	16 58 40·38	−16 3 37·98
	7 Ophiuchi	2·6	17 4 59·14	−15 36 32·12
b	−15°, 4502	6·7	17 10 53·66	−15 7 6·94
5 *a*	+12°, 3234	6·7	17 25 59·98	+11 59 45·78
	α Ophiuchi	2·1	17 30 34·24	+12 37 40·72
b	+13°, 3421	6·3	17 34 38·73	+13 22 51·73
6 *a*	−9°, 4616	8·9	17 46 49·71	− 9 57 13·63
	ν Ophiuchi	3·5	17 53 51·07	− 9 45 44·99
b	−9°, 4646	8·6	18 0 0·97	− 9 35 0·23
7 *a*	−3°, 4255	9·5	18 8 38·83	− 3 0 44·65
	7 Serpentis	3·5	18 16 26·72	− 2 55 24·93
b	−2°, 4638	8·3	18 24 20·45	− 2 50 0·69

TABLE I.—*continued.*

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0.	Dec.
			h m s	° ′ ″
8 *a*	−4°, 4557	9·2	18 37 23·90	− 4 54 0·99
	6 Scuti	4·5	18 42 11·22	− 4 50 56·03
b	−4°, 4603	9·2	18 47 50·28	− 4 50 38·87
9 *a*	−5°, 4835	9·1	18 55 25·09	− 5 30 7·85
	λ Aquilæ	3·5	19 1 15·63	− 5 1 25·81
b	−4°, 4719	8·6	19 7 23·99	− 4 37 11·97
10 *a*	−5°, 4845	8·3	18 57 50·51	− 5 40 17·73
	λ Aquilæ	3·5	19 1 15·63	− 5 1 25·81
b	−4°, 4712	8·7	19 5 15·79	− 4 12 10·57
11 *a*	+2°, 3856	7·8	19 15 55·70	+ 2 45 43·22
	δ Aquilæ	3·4	19 20 45·54	+ 2 55 36·96
b	+3°, 4043	6·4	19 25 50·94	+ 3 14 51·72
12 *a*	+3°, 3990	8·8	19 17 42·36	+ 3 9 33·36
	δ Aquilæ	3·4	19 20 45·54	+ 2 55 36·96
b	+2°, 3892	5·9	19 23 37·64	+ 2 44 18·32
13 *a*	+8°, 4198	8·4	19 39 50·81	+ 8 51 55·66
	α Aquilæ	0·8	19 46 11·83	+ 8 37 10·64
b	+8°, 4275	8·0	19 52 21·75	+ 8 11 48·34
14 *a*	+8°, 4227	9·0	19 44 39·25	+ 8 22 10·96
	α Aquilæ	0·8	19 46 11·83	+ 8 37 10·64
b	+8°, 4247	8·2	19 47 57·91	+ 8 53 20·02
15 *a*	−0°, 3911	8·6	20 2 1·01	− 0 24 11·00
	θ Aquilæ	3·4	20 6 27·31	− 1 6 2·36
b	−1°, 3935	7·5	20 11 34·11	− 1 47 16·90
16 *a*	−1°, 3902	8·0	20 4 31·69	− 1 32 57·82
	θ Aquilæ	3·4	20 6 27·31	− 1 6 2·36
b	−0°, 3942	7·5	20 8 14·67	− 0 36 47·98
17 *a*	−18°, 5663	8·5	20 18 10·75	−18 31 1·02
	π Capricorni	5·1	20 21 56·51	−18 31 12·58
b	−18°, 5705	8·3	20 26 2·67	−18 24 0·52
18 *a*	−5°, 5349	7·9	20 37 31·64	− 5 38 0·18
	3 Aquarii	4·5	20 42 46·70	− 5 22 20·10
b	−5°, 5402	8·5	20 47 21·78	− 5 3 22·24

TABLE I.—*continued.*

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0. h m s	Dec. ° ′ ″
19 *a*	−5°, 5368	7·7	20 38 58·33	− 5 55 44·88
	3 Aquarii	4·5	20 42 46·70	− 5 22 20·10
b	−5°, 5395	8·1	20 45 45·03	− 5 8 30·84
20 *a*	−17°, 6140	8·3	20 55 10·77	−17 14 39·95
	θ Capricorni	4·1	21 0 39·87	−17 36 24·39
b	−18°, 5886	8·3	21 8 31·41	−17 57 9·39
21 *a*	−17°, 6167	7·6	20 59 34·81	−17 32 13·51
	θ Capricorni	4·1	21 0 39·87	−17 36 24·39
b	−18°, 5862	6·0	21 2 27·97	−17 49 58·95
22 *a*	−6°, 5757	8·3	21 21 37·93	− 6 24 29·01
	β Aquarii	3·0	21 26 36·68	− 5 59 6·11
b	−5°, 5592	7·7	21 31 36·79	− 5 38 22·65
23 *a*	−6°, 5761	8·9	21 23 5·00	− 6 1 43·73
	β Aquarii	3·0	21 26 36·68	− 5 59 6·11
b	−6°, 5782	8·8	21 28 57·14	− 5 53 47·51
24 *a*	+9°, 4871	8·9	21 35 27·07	+ 9 33 42·93
	ε Pegasi	2·7	21 39 34·15	+ 9 26 37·45
b	+9°, 4899	8·7	21 43 23·01	+ 9 24 36·95
25 *a*	+5°, 4947	7·7	22 0 58·56	+ 5 30 32·09
	θ Pegasi	3·7	22 5 27·48	+ 5 44 6·65
b	+5°, 4982	8·5	22 10 51·38	+ 6 10 29·60
26 *a*	−11°, 5823	8·0	22 19 31·52	−11 37 58·19
	σ Aquarii	4·8	22 25 40·44	−11 9 32·79
b	−11°, 5875	8·9	22 31 41·51	−10 46 31·29
27 *a*	−14°, 6337	8·6	22 39 59·06	−14 7 52·55
	τ Aquarii	4·4	22 44 36·92	−14 5 21·01
b	−14°, 6367	8·9	22 50 25·35	−14 14 28·93
28 *a*	+15°, 4737	9·0	22 54 12·54	+15 16 43·73
	α Pegasi	2·6	23 0 4·66	+14 41 57·77
b	+13°, 5059	7·8	23 5 16·14	+13 55 7·89
29 *a*	−20°, 6568	8·7	23 11 40·04	−20 28 29·33
	b′ Aquarii	4·3	23 18 2·08	−20 36 49·97
b	−20°, 6606	8·8	23 25 9·70	−20 41 22·09

TABLE I.—*continued.*

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0. h m s	Dec. 1906·0. ° ′ ″
30 *a*	−15°, 6462	8·5	23 32 32·46	−14 44 13·17
	ω^2 Aquarii	4·5	23 37 50·88	−15 3 53·03
b	−15°, 6500	8·9	23 43 44·24	−15 22 50·19
31 *a*	+5°, 5230	8·4	23 48 19·79	+ 6 10 34·47
	ω Piscium	4·0	23 54 29·03	+ 6 20 34·66
b	+6°, 5242	7·8	0 2 15·15	+ 6 21 10·58
32 *a*	−9°, 23	8·5	0 7 33·83	− 9 13 56·19
	i Ceti	3·7	0 14 38·33	− 9 20 41·89
b	−9°, 79	8·0	0 22 33·65	− 9 10 40·25
33 *a*	−18°, 98	8·8	0 31 33·79	−18 7 41·30
	β Ceti	2·3	0 38 52·31	−18 30 8·60
b	−19°, 133	8·6	0 45 59·89	−19 1 2·22

The results of the observations are contained in the following Tables II. and III.

The transit observations have been made by seven different observers, two using the old Transit Circle and recording the times of transit over fixed wires in the usual manner by means of a chronograph, and the remaining five observing with the new reversible Transit Circle by means of a travelling wire guided by hand, and automatically recording on the chronograph the instants when the wire reached certain fixed positions. The results have been referred to the equinox 1906·0 by the application of the usual star-corrections. Table II. contains the observed differences of R.A. derived by the different observers after application of these corrections, the suffixes indicating the number of separate observations involved in each result.

The heliometer measures have been made by two observers, Messrs Whittingdale and Baldwin. The results quoted in Table III. are the mean results from all the measures of each pair. They have been corrected for refraction, but, except in a few cases, no corrections for aberration or nutation have been applied, as these quantities will be practically eliminated simultaneously with the instrumental scale-constant and index correction to the position-circle

TABLE II.

Observed Differences of Right Ascensions.

Epoch 1906·0.

Instrument.	8-inch Transit.		Reversible Transit Circle.				
Observer.	Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.	Jackson.
	m s	m s	m s	m s	m s	m s	m s
1 *a*	...	4 44·075$_2$	4 44·135$_1$	...	...	...	...
b	...	3 23·692$_2$	3 23·597$_1$	...	...	...	...
2 *a*	...	5 36·185$_2$	5 36·230$_2$	...	5 36·225$_1$	...	...
b	...	5 50·015$_2$	5 49·970$_2$	...	5 49·935$_1$	...	...
3 *a*	...	4 27·514$_3$	4 27·564$_1$	...	...	4 27·524$_1$	4 27·614$_1$
b	...	3 44·178$_3$	3 44·141$_1$	...	...	3 44·151$_1$	3 44·081$_1$
4 *a*	6 18·694$_2$	6 18·647$_3$	6 18·679$_2$	...	6 18·709$_2$	...	6 18·724$_1$
b	5 54·470$_2$	5 54·557$_3$	5 54·512$_2$	...	5 54·472$_2$	...	5 54·465$_1$
5 *a*	4 34·146$_2$	4 34·168$_4$	4 34·201$_2$	...	4 34·279$_1$	4 34·259$_1$	4 34·269$_1$
b	4 4·520$_2$	4 4·508$_4$	4 4·485$_2$	...	4 4·455$_1$	4 4·455$_1$	4 4·435$_1$
6 *a*	7 1·217$_1$	7 1·197$_3$	7 1·270$_2$	7 1·247$_1$	7 1·242$_1$	7 1·310$_1$	7 1·310$_1$
b	6 9·909$_1$	6 9·945$_3$	6 9·873$_2$	6 9·899$_1$	6 9·917$_1$	6 9·848$_1$	6 9·848$_1$
7 *a*	...	7 47·742$_5$	7 47·811$_2$	7 47·762$_1$	7 47·829$_1$	...	7 47·848$_1$
b	...	7 53·716$_5$	7 53·691$_2$	7 53·722$_1$	7 53·670$_1$	...	7 53·619$_1$
8 *a*	4 47·277$_1$	4 47·196$_4$	4 47·281$_2$	...	4 47·263$_1$	...	4 47·312$_1$
b	5 39·054$_1$	5 39·088$_4$	5 39·048$_2$	...	5 39·010$_1$	...	5 38·979$_1$
9 *a*	5 50·477$_3$	5 50·447$_6$	5 50·507$_4$	5 50·526$_2$	5 50·564$_1$	...	5 50·489$_2$
b	6 8·389$_3$	6 8·416$_6$	6 8·356$_4$	6 8·351$_2$	6 8·313$_1$	...	6 8·319$_2$
10 *a*	3 25·144$_4$	3 25·122$_3$	...	...	...	...	...
b	4 0·112$_4$	4 0·167$_3$	...	...	...	...	...
11 *a*	4 49·789$_4$	4 49·739$_7$	4 49·819$_3$	4 49·823$_2$	4 49·795$_3$	4 49·846$_1$	4 49·834$_3$
b	5 5·322$_4$	5 5·403$_7$	5 5·356$_3$	5 5·333$_2$	5 5·365$_3$	5 5·356$_1$	5 5·338$_3$
12 *a*	3 3·194$_2$	3 3·162$_5$	...	...	...	...	...
b	2 52·048$_2$	2 52·099$_5$	...	...	...	...	...
13 *a*	6 20·946$_6$	6 20·913$_5$	6 20·956$_1$	6 20·946$_1$	6 21·017$_2$	6 21·014$_2$	6 20·972$_1$
b	6 9·867$_6$	6 9·870$_5$	6 9·876$_1$	6 9·785$_1$	6 9·776$_2$	6 9·803$_2$	6 9·893$_1$
14 *a*	1 32·548$_4$	1 32·537$_5$	...	...	...	...	...
b	1 46·042$_4$	1 46·082$_5$	...	...	...	...	...
15 *a*	4 26·223$_4$	4 26·251$_4$	4 26·284$_1$	4 26·251$_3$	4 26·236$_2$	4 26·279$_1$	4 26·298$_3$
b	5 6·768$_4$	5 6·759$_4$	5 6·761$_1$	5 6·775$_3$	5 6·743$_2$	5 6·758$_1$	5 6·739$_3$
16 *a*	1 55·632$_4$	1 55·639$_4$	...	..	...	...	...
b	1 47·332$_4$	1 47·352$_4$	...	...	...	...	...

TABLE II.—*continued.*

Instrument.		8-inch Transit.		Reversible Transit Circle.				
Observer.		Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.	Jackson.
		m s	m s	m s	m s	m s	m s	m s
17	*a*	3 45·821$_4$	3 45·787$_3$	3 45·816$_1$	3 45·761$_2$	3 45·812$_2$	3 45·777$_2$	3 45·781$_3$
	b	4 6·091$_4$	4 6·104$_3$	4 6·086$_1$	4 6·141$_2$	4 6·102$_2$	4 6·148$_2$	4 6·141$_3$
18	*a*	5 15·029$_4$	5 15·023$_3$	...	5 15·023$_4$	5 15·047$_3$	5 15·058$_3$	5 15·045$_2$
	b	4 35·123$_4$	4 35·122$_3$	...	4 35·097$_4$	4 35·059$_3$	4 35·076$_3$	4 35·087$_2$
19	*a*	3 48·363$_4$	3 48·357$_3$	...	...	...	...	...
	b	2 58·309$_4$	2 58·338$_3$	...	...	...	...	...
20	*a*	6 29·125$_1$	6 29·129$_1$	6 29·078$_1$	6 29·183$_2$	6 29·111$_3$	6 29·116$_5$	6 29·149$_2$
	b	7 51·533$_1$	7 51·524$_1$	7 51·522$_1$	7 51·506$_2$	7 51·510$_3$	7 51·517$_5$	7 51·482$_2$
21	*a*	1 5·031$_4$	1 5·023$_3$	...	...	...	...	...
	b	1 48·093$_4$	1 48·087$_3$	...	...	...	...	...
22	*a*	4 58·727$_5$	4 58·755$_1$	4 58·725$_1$	4 58·752$_4$	4 58·745$_2$	4 58·746$_6$	4 58·757$_3$
	b	5 0·103$_5$	5 0·077$_1$	5 0·127$_1$	5 0·124$_4$	5 0·063$_2$	5 0·124$_6$	5 0·110$_3$
23	*a*	3 31·661$_5$	3 31·578$_1$	...	...	...	...	...
	b	2 20·476$_5$	2 20·472$_1$	...	...	...	...	...
24	*a*	4 7·040$_4$	4 6·987$_1$	4 7·017$_1$	4 7·024$_3$	4 7·077$_2$	4 7·086$_3$	4 7·044$_2$
	b	3 48·891$_4$	3 48·918$_1$	3 48·858$_1$	3 48·872$_3$	3 48·843$_2$	3 48·844$_3$	3 48·865$_2$
25	*a*	4 28·907$_3$	4 28·878$_1$	4 28·858$_2$	4 28·907$_3$	4 28·870$_3$	4 28·886$_4$	4 28·907$_4$
	b	5 23·885$_3$	5 23·973$_1$	5 23·913$_2$	5 23·897$_3$	5 23·925$_3$	5 23·918$_4$	5 23·894$_4$
26	*a*	6 8·872$_2$	...	6 8·880$_1$	6 8·901$_2$	6 8·903$_2$	6 8·918$_3$	6 8·923$_3$
	b	6 1·094$_2$	...	6 1·012$_1$	6 1·079$_2$	6 1·040$_2$	6 1·037$_3$	6 1·078$_3$
27	*a*	4 37·844$_1$	4 37·796$_1$	4 37·875$_1$	4 37·846$_1$	4 37·824$_1$	4 37·875$_2$	4 37·849$_3$
	b	5 48·458$_1$	5 48·531$_1$	5 48·443$_1$	5 48·437$_1$	5 48·468$_1$	5 48·434$_2$	5 48·390$_3$
28	*a*	5 52·140$_2$	...	5 52·109$_2$	5 52·164$_1$	5 52·072$_2$	5 52·123$_3$	...
	b	5 11·474$_2$	...	5 11·473$_2$	5 11·478$_1$	5 11·506$_2$	5 11·453$_3$	...
29	*a*	6 22·070$_1$	6 21·909$_1$	6 22·031$_2$	6 21·984$_1$	6 22·049$_3$	6 22·021$_3$	6 22·004$_3$
	b	7 7·631$_1$	7 7·670$_1$	7 7·618$_2$	7 7·630$_1$	7 7·603$_3$	7 7·589$_3$	7 7·637$_3$
30	*a*	5 18·536$_1$	5 18·395$_1$	5 18·458$_2$	5 18·393$_1$	5 18·426$_3$	5 18·480$_3$	5 18·395$_3$
	b	5 53·125$_1$	5 53·394$_1$	5 53·313$_2$	5 53·375$_1$	5 53·339$_3$	5 53·309$_3$	5 53·345$_3$
31	*a*	6 9·257$_1$	6 9·106$_1$	6 9·236$_2$	6 9·266$_1$	6 9·234$_3$	6 9·258$_3$	6 9·241$_3$
	b	7 46·215$_1$	7 46·213$_1$	7 46·099$_2$	7 46·099$_1$	7 46·191$_3$	7 46·094$_3$	7 46·112$_3$
32	*a*	7 4·385$_1$	...	7 4·528$_1$	...	7 4·506$_3$	7 4·490$_8$	7 4·517$_2$
	b	7 55·441$_1$	...	7 55·280$_1$	...	7 55·295$_3$	7 55·310$_3$	7 55·331$_2$
33	*a*	...	...	7 18·515$_2$	7 18·576$_1$	7 18·537$_3$	7 18·559$_3$	7 18·573$_1$
	b	...	...	7 7·585$_2$	7 7·586$_1$	7 7·642$_3$	7 7·572$_3$	7 7·595$_1$

TABLE III.

Table of Instrumental Distances and Position-Angles observed with the Heliometer.

Group.	No. of Observation. D.	P.A.	a. Distance.	a. Position-Angle.	b. Distance.	b. Position-Angle.
1	3	2	4289″·419	277° 59′ 48″	3299″·816	293° 7′ 31″
2	3	1	4569·911	278 23 13	4711·909	273 9 33
3	2	1	3971·132	277 9 18	3304·070	269 26 39
4	3	2	5702·217	254 0 19	5422·158	251 34 51
5	2	2	4617·319	241 3 32	4483·717	233 23 13
6	3	1	6262·879	264 14 24	5506·352	263 49 53
7	3	1	7015·268	267 58 24	7102·951	267 57 26
8	3	1	4297·166	268 7 12	5067·526	271 44 11
9	3	2	5510·700	252 22 22	5693·633	255 46 51
10	3	3	3849·927	233 18 15	4650·005	231 6 24
11	4	2	4381·633	262 47 1	4716·047	256 23 16
12	3	2	2868·977	287 32 12	2665·012	285 18 48
13	3	2	5716·171	279 28 13	5693·492	286 4 18
14	3	3	1642·636	237 21 33	1845·856	238 55 22
15	3	3	4717·547	302 44 44	5222·509	298 51 26
16	3	3	2369·918	227 36 35	2380·705	223 6 48
17	3	1	3211·763	270 46 36	3528·059	263 31 31
18	3	2	4796·276	259 16 9	4263·023	255 4 48
19	3	3	3954·919	240 6 58	2789·282	253 14 37
20	3	2	5719·491	283 45 15	6848·518	281 2 45
21	3	3	963·525	285 42 17	1745·713	298 23 20
22	3	2	4708·532	251 41 55	4843·676	248 12 41
23	3	1	3161·216	267 42 56	2120·031	261 56 20
24	3	1	3679·683	277 12 26	3388·578	272 36 47
25	3	2	4095·903	259 6 58	5085·403	252 26 28
26	3	2	5686·703	253 7 15	5493·288	256 0 56
27	3	1	4045·352	268 24 42	5097·784	276 46 35
28	3	3	5511·906	292 48 46	5327·954	302 24 6
29	3	1	5389·438	275 53 44	6008·341	273 10 11
30	3	2	4764·892	284 54 45	5239·097	283 7 28
31	3	1	5538·700	264 21 44	6949·113	270 16 35
32	3	1	6297·759	274 15 11	7062·612	265 40 51
33	3	2	6389·479	282 45 16	6349·276	287 32 53

Reduction of Heliometer Observations.

The approximate places of the stars for the epoch 1906·0 are quoted in Table I. With these approximate places, tabular distances and position-angles were computed. On comparing these tabular distances with the distances as derived from the heliometer measures we obtain relations between the corrections to the co-ordinates of the stars of the following form,—

$$S_a\sigma + (\Delta\alpha_2 - \Delta\alpha_1)\cos\delta_a \sin p_a + (\Delta\delta_2 - \Delta\delta_1)\cos p_a = O_a - C_a$$
$$S_b\sigma + (\Delta\alpha_3 - \Delta\alpha_2)\cos\delta_b \sin p_b + (\Delta\delta_3 - \Delta\delta_2)\cos p_b = O_b - C_b$$

where the suffixes 1, 2, 3 refer to the stars in increasing order of R.A., the suffixes a, b to the pairs 1–2, 2–3 respectively. S denotes the distance, σ a constant depending on the instrumental scale-value, δ the mean declination, and p the position-angle of a pair, and O, C respectively the observed and computed distances.

On eliminating σ we derive

$$(\Delta\alpha_2 - \Delta\alpha_1)\frac{\cos\delta_a \sin p_a}{S_a} - (\Delta\alpha_3 - \Delta\alpha_2)\frac{\cos\delta_b \sin p_b}{S_b}$$
$$+ (\Delta\delta_2 - \Delta\delta_1)\frac{\cos p_a}{S_a} - (\Delta\delta_3 - \Delta\delta_1)\frac{\cos p_b}{S_b} = \frac{O_a - C_a}{S_a} - \frac{O_b - C_b}{S_b} \quad \ldots \text{ I.}$$

In like manner, from the observations of position-angle introducing a constant quantity π to denote the mean index correction to the position-circle during the several observations of a group, we form equations of condition as follows,—

$$S_a\pi + (\Delta\alpha_2 - \Delta\alpha_1)\cos\delta_a \cos p_a - (\Delta\delta_2 - \Delta\delta_1)\sin p_a = (O_a' - C_a')\sin S_a$$
$$S_b\pi + (\Delta\alpha_3 - \Delta\alpha_2)\cos\delta_b \cos p_b - (\Delta\delta_3 - \Delta\delta_2)\sin p_b = (O_b' - C_b')\sin S_b$$

and on elimination of π, S_a, S_b being expressed in seconds of arc

$$(\Delta\alpha_2 - \Delta\alpha_1)\frac{\cos\delta_a \cos p_a}{S_a} - (\Delta\alpha_3 - \Delta\alpha_2)\frac{\cos\delta_b \cos p_b}{S_b}$$
$$- (\Delta\delta_2 - \Delta\delta_1)\frac{\sin p_a}{S_a} + (\Delta\delta_3 - \Delta\delta_2)\frac{\sin p_b}{S_b} = (O_a' - C_a' - O_b' + C_b')\sin 1'' \quad \ldots \text{ II.}$$

On eliminating $\Delta\delta_2$ from the equations I. and II. we obtain a linear relation between $\Delta\alpha_1$, $\Delta\alpha_2$, $\Delta\alpha_3$, $\Delta\delta_1 - \Delta\delta_3$. The algebraic elimination is cumbersome, but the numerical elimination is easily performed in special cases. This relation takes the form

$$\alpha(\Delta\alpha_2 - \Delta\alpha_1) - \beta(\Delta\alpha_3 - \Delta\alpha_2) = \gamma(\Delta\delta_1 - \Delta\delta_3) + n$$

where, if the stars selected are well chosen, we have approximately $\alpha = \beta$, while γ is small in comparison with either of them. In forming these equations such a factor has been introduced as to reduce the coefficient of $\Delta\alpha_2$ to unity, *i.e.* so that $\alpha + \beta = 1$.

The following are the equations of condition resulting in this manner from the heliometer measures:—

Equations of Condition resulting from Heliometer Measures.

Group.				
1	$0\cdot433(\Delta\alpha_2-\Delta\alpha_1)$	$-0\cdot566(\Delta\alpha_3-\Delta\alpha_2)$	$=+0\cdot065(\Delta\delta_1-\Delta\delta_3)$	$+0''\cdot29$
2	0·508	−0·492	= −0·025	+0·53
3	0·455	−0·546	= −0·035	+0·33
4	0·487	−0·514	= −0·011	+0·25
5	0·493	−0·506	= −0·035	+0·17
6	0·468	−0·532	= −0·002	−0·43
7	0·503	−0·497	= 000	+0·42
8	0·541	−0·459	= +0·005	−0·03
9	0·507	−0·492	= +0·015	−0·04
10	0·547	−0·453	= −0·009	+0·63
11	0·519	−0·481	= −0·028	+0·44
12	0·482	−0·518	= −0·009	+0·70
13	0·499	−0·501	= +0·029	+0·65
14	0·529	−0·470	= +0·007	+1·07
15	0·527	−0·475	= −0·017	+0·23
16	0·501	−0·499	= −0·020	+0·11
17	0·524	−0·476	= −0·034	+0·85
18	0·470	−0·530	= −0·018	+0·10
19	0·412	−0·587	= +0·056	+0·96
20	0·545	−0·454	= −0·012	+0·49
21	0·647	−0·353	= +0·053	+0·01
22	0·507	−0·493	= −0·015	+0·01
23	0·401	−0·599	= −0·024	−0·44
24	0·480	−0·521	= −0·020	−0·09
25	0·553	−0·446	= −0·029	−0·30
26	0·491	−0·509	= +0·013	+0·29
27	0·559	−0·441	= +0·037	0·00
28	0·491	−0·509	= +0·043	+0·12
29	0·527	−0·473	= −0·013	−0·03
30	0·524	−0·475	= −0·008	+0·69
31	0·556	−0·443	= +0·026	+0·27
32	0·529	−0·471	= −0·038	+0·32
33	0·499	−0·501	= +0·020	+0·68

The quantities $\Delta\delta_1$, $\Delta\delta_3$ are retained in the right-hand members, as in a few cases the finally adopted values of the declinations differ from those quoted in Table I., which were derived from a few early observations only. The following are the corrections derived from the inclusion of additional observations of the declination with the Transit Circles :—

Group.	$\Delta\delta_1$	$\Delta\delta_3$
	″	″
3	−0·38	...
18	− ·39	−0·51
19	− ·06	−1·05
20	− ·16	− ·11
21	− ·50	− ·30
28	...	+ ·30
32	− ·13	− ·68
33	...	− ·26

In order to subject the magnitude personality to an analytical treatment, it was now assumed that each transit observation is affected by an error which may be expressed analytically by the formula

$$\alpha(m-4)+\beta(m-4)^2$$

where m denotes the magnitude of the star and α, β constants for the observer. The adopted magnitudes are those quoted in Table I.

Thus for any pair of stars of R.A. α_1, α_2, say of magnitudes m_1, m_2, the observed difference of R.A. may be equated to

$$\alpha_2-\alpha_1+\alpha(m_2-m_1)+\beta(m_2-m_1)(m_2+m_1-8)\,;$$

or if we denote this quantity by O and the tabular difference of R.A. as dependent on Table I. by C we find

$$\Delta\alpha_2-\Delta\alpha_1=\mathrm{O}-\mathrm{C}-[\alpha(m_2-m_1)+\beta(m_2-m_1)(m_2+m_1-8)]$$

On substituting these expressions in the left-hand members of the equations of condition resulting from the heliometer measures, we derive a series of linear equations for the determination of the quantities of α, β which may be expressed in the following tabular form. The suffixes attached to the absolute terms represent the number of meridian observations involved in each.

Group.	Coefficients of		Absolute terms for						
	α	β	Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.	Jackson.
			″	″	″	″	″	″	″
1	5·31	18·73	...	$+0''.78_2$	$-0''.43_1$	...	...	...	...
2	7·30	13·92	..	$+1''.07_2$	$+0''.39_2$	...	$+0''.18_1$	...	...
3	3·61	18·02	...	$+1''.08_3$	$+0''.44_1$	...	...	$+0''.78_1$	$-0''.41_1$
4	5·07	12·50	$+0''.34_2$	$+1''.37_3$	$+0''.78_2$	...	$+0''.25_2$	...	$+0''.08_1$
5	4·89	3·08	$+1''.16_2$	$+0\ 91_4$	$+0''.49_2$	...	$-0''.31_1$	$-0''.16_1$	$-0''.40_1$
6	5·30	22·74	$+0''.65_1$	$+1''.08_3$	$-0''.02_2$	$+0''.36_1$	$+0''.54_1$	$-0''.49_1$	$-0''.49_1$
7	5·41	24·14	...	$+1''.28_5$	$+0''.57_2$	$+1''.18_1$	$+0''.29_1$	...	$-0''.24_1$
8	4·70	26·80	$+0''.28_1$	$+1''.17_4$	$+0''.20_2$	...	$+0''.07_1$	...	$-0''.53_1$
9	5·40	23·25	$+0''.66_1$	$+1''.08_6$	$+0''.18_4$	$0''.00_2$	$-0''.56_1$	...	$+0''.04_2$
10	5·03	19·84	$+0''.10_4$	$+0''.66_3$	...	...	...	...	...
11	3·72	9·92	$+0''.27_4$	$+1''.25_7$	$+0''.29_3$	$+0''.09_2$	$+0''.54_3$	$+0''.07_1$	$+0''.04_3$
12	3·85	12·45	$+0''.20_2$	$+0''.82_5$	...	...	...	...	...
13	7·30	7·35	$+0''.80_6$	$+1''.07_5$	$+0''.80_1$	$+0''.19_1$	$-0''.41_2$	$-0''.19_2$	$+0''.81_1$
14	7·82	12·06	$+1''.05_4$	$+1''.42_5$	..	...	...	...	...
15	4·74	16·50	$+0''.61_4$	$+0''.33_4$	$+0''.08_1$	$+0''.44_3$	$+0''.34_2$	$+0''.10_1$	$-0''.19_3$
16	4·35	13·55	$-0''.19_4$	$-0''.10_4$	...	...	...	...	...
17	3·30	18·34	$-0''.13_4$	$+0''.24_3$	$-0''.12_1$	$+0''.70_8$	$+0''.03_2$	$+0''.62_3$	$+0''.55_3$
18	3·67	17·42	$+0''.66_4$	$+0''.69_3$	...	$+0''.50_4$	$+0''.02_3$	$+0''.08_3$	$+0''.27_2$
19	3·39	15·10	$+0''.72_4$	$+1''.02_3$	...	...	...	...	...
20	4·19	18·48	$+0''.23_1$	$+0''.14_1$	$+0''.55_1$	$-0''.43_2$	$+0''.20_3$	$+0''.20_5$	$-0''.30_2$
21	2·93	9·82	$+0''.24_4$	$+0''.29_3$	...	...	...	...	...
22	5·01	15·13	$+0''.13_5$	$-0''.27_1$	$+0''.33_1$	$+0''.10_4$	$-0''.29_2$	$+0''.15_6$	$-0''.03_3$
23	5·83	22·40	$-0''.18_5$	$+0''.28_1$	...	...	...	...	...
24	6·20	21·68	$+0''.44_4$	$+1''.03_1$	$+0''.35_1$	$+0''.40_3$	$-0''.21_2$	$-0''.26_3$	$+0''.21_2$
25	4·35	16·31	$-0''.29_3$	$+0''.54_1$	$+0''.30_2$	$-0''.21_3$	$+0''.28_3$	$+0''.10_4$	$-0''.23_4$
26	3·71	19·67	$+0''.75_2$	...	$+0''.06_1$	$+0''.42_2$	$+0''.11_2$	$-0''.03_3$	$+0''.25_3$
27	4·38	22·56	$+0''.25_1$	$+1''.14_1$	$-0''.11_1$	$+0''.10_1$	$+0''.49_1$	$-0''.17_2$	$-0''.23_3$
28	5·84	17·60	$-0''.09_2$	...	$+0''.13_2$	$-0''.23_1$	$+0''.66_2$	$-0''.11_3$	...
29	4·45	22·47	$-0''.19_1$	$+1''.33_1$	$+0''.03_2$	$+0''.48_1$	$-0''.22_3$	$-0''.10_3$	$+0''.37_3$
30	4·14	21·71	$-1''.89_1$	$+1''.13_1$	$-0''.05_2$	$+1''.01_1$	$+0''.49_3$	$-0''.15_3$	$+0''.78_3$
31	4·18	17·26	$+0''.76_1$	$+2''.01_1$	$+0''.16_2$	$-0''.09_1$	$+0''.79_3$	$-0''.05_3$	$+0''.21_3$
32	4·57	18·18	$+2''.08_1$	...	$-0''.20_1$	...	$+0''.07_3$	$+0''.31_3$	$+0''.24_2$
33	6·40	19·55	...	...	$+0''.76_2$	$+0''.31_1$	$+1''.02_3$	$+0''.33_3$	$+0''.40_1$

For the purpose of combining these equations, it was assumed that the errors in the absolute terms resulting from the heliometer measures were insignificant compared with those resulting from the transit observations, and the equations were accordingly weighted according to the number of separate observations of each group

obtained by the meridian circle observers. A least square solution then led to the following results:—

Observer.	α.		β.		Probable error of	
	In arc.	In time.	In arc.	In time.	α.	β.
	"	s	"	s	s	s
Power .	+0·134	+0·0089	−0·020	−0·0013	±0·0020	±0·0006
Pead .	+ ·148	+ ·0099	+ ·011	+ ·0007	± ·0016	± ·0005
Wilkin .	+ ·117	+ ·0078	− ·018	− ·0012	± ·0012	± ·0003
Jeffries .	− ·053	− ·0035	+ ·028	+ ·0019	± ·0027	± ·0007
Mullis .	+ ·011	+ ·0007	+ ·010	+ ·0007	± ·0023	± ·0006
Wood .	− ·015	− ·0010	+ ·006	+ ·0004	± ·0016	± ·0004
Jackson .	− ·003	− ·0002	+ ·006	+ ·0004	± ·0025	± ·0006

The resulting corrections to the observations to make all correspond with those of stars of the fourth magnitude are as follows:—

Correction for Personal Equation in R.A. depending on Magnitude.

Mag.	Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.	Jackson.
	s	s	s	s	s	s	s
0	+0·057	+0·029	+0·050	−0·044	−0·007	−0·010	−0·007
1	38	24	34	27	− 4	7	− 4
2	23	17	21	14	− 1	4	− 2
3	+ ·010	+ ·009	+ ·009	− ·005	000	− ·001	− ·001
4	...	...	...	...	...	..	...
5	− ·008	− ·011	− ·007	+ ·002	− ·002	− ·001	000
6	− 13	23	11	000	− 4	000	− ·001
7	− 16	36	12	− ·006	− 8	− ·001	− ·003
8	− 15	51	12	− 16	− 13	− ·002	− ·006
9	− 14	− ·068	− ·009	− ·029	− 20	− ·005	− ·009

The quantities thus derived in the case of the observers who used the travelling wire method appear to be quite insignificant except at the extreme limits of magnitude involved in the table. The strength of the determination at these limits depends, however, not so much on the actual weight derived directly from the observations, as on the weight artificially extended to it by the assumption that the magnitude personality may be expressed by the formula $am + \beta m^2$.

The use of such a formula can only be justified as a convenient means of interpolation over the range of magnitude covered by the stars actually observed. If we limit ourselves to the range $3 - 8$, within which limits the majority of the stars are contained, we may safely conclude that the observations dealt with give no evidence of the existence of sensible personality depending on magnitude. Beyond these limits the quantities are not larger than might reasonably be expected to arise from the use of an

empirical 'extrapolation' formula, which will have the effect of magnifying the accidental errors of observation at the extremities of the table.

The results for the observers with the old Transit Circle from observations made partly through screens in 1900 and 1904 are as follows:—

Mag.	Power.	Pead.
	s	s
0	+0·008	+0·037
1	7	28
2	5	18
3	+0·003	+ ·009
4	...	...
5	− ·004	− ·009
6	− 9	− ·018
7	− 15	− ·026
8	− 21	− ·034
9	− 28	− ·042

The agreement with the present results is as close as could be expected, except in the case of the brighter stars for Power and the fainter stars for Pead. The discordances are doubtless due to the inadequacy of the material employed to correctly determine the magnitude personality at these extreme limits of magnitude, rather than to any real change in the observer's habits during the interval between the two sets of observations.

The plan of these observations was prepared by Sir David Gill, and the observations were well advanced before his departure from the Cape. The observations were completed and the computations made and prepared for press in their present form under the direction of Mr S. S. Hough.

On the Value of the Solar Parallax resulting from the Greenwich Photographs of Eros, 1900–1901.

(*Communicated by the Astronomer Royal.*)

The discussion of the photographs of Eros taken at Greenwich during the opposition of 1900–1901 has now been completed, and a value of the solar parallax has been deduced as resulting from the photographs taken at Greenwich. The plan of the Eros Commission contemplated, ultimately, the combined discussion of the photographs taken at all the co-operating observatories, but there is obviously great difficulty in treating such a mass of heterogeneous material, and it seems desirable, in the first instance, to discuss separately the observations of the individual observatories, and deduce a value of the solar parallax in each case.

With this view the results obtained from the Greenwich photographs are here given. There were in all 197 photographs taken with the Astrographic 13-inch refractor and 153 taken with the Thompson 26-inch refractor between 1900 October 1 and 1901 February 25, but for the determination of the solar parallax the discussion has been confined to the period from 1900 October 14 to 1901 January 18 (151 photographs with the Astrographic and 103 with the Thompson), the material before and after these dates not being suitable for determination of the solar parallax, though useful for the position of Eros.

It may be recalled here that, as explained in a paper on the "New Greenwich micrometer for measurement of photographs of Eros" (*Monthly Notices*, vol. lxiv. p. 633), ten to twelve "reference" stars and six "comparison" stars were measured with Eros on each Astrographic photograph, and the results from these two sets of stars to which Eros was referred have been discussed separately. The "reference" stars selected from M. Loewy's list within 55′ of the plate centre are all brighter than ninth magnitude and therefore brighter than Eros. The "comparison" stars, on the other hand, are of approximately the same brightness as the planet and within 25′ of the plate centre, so as to be well within the field of the Thompson plates for which they served as points of reference.

The determination of the movement of Eros in the interval between the groups of photographs compared, which is essential for the deduction of the parallax, was first undertaken. The measured positions of Eros were compared with tabular places from M. Millosevich's ephemerides (Astrographic Conference, Circular No. 9), using the value 8″·800 for the solar parallax, which was a sufficient approximation for this purpose. The change of error of the ephemeris during the interval between the groups of observations to be compared for parallax was generally considerable (amounting at times to ″·10 per day), but it could be determined with great accuracy. The means of groups of measured positions of Eros were taken so as to give the error of the ephemeris at intervals of about five days; from these the error of the ephemeris was represented by a smooth curve, which represented the observations very satisfactorily.

The correction to the adopted parallax was derived by comparing observations at different hour-angles. It was not usually desirable to make comparisons between groups of observations separated by an interval of more than one day, owing to the large accidental errors introduced by erroneous places of the stars to which Eros was referred. The same reference and comparison stars were used throughout each night, so that in deducing the parallax from comparisons between photographs taken on an evening and the following morning errors in the star-places are eliminated; but in all other comparisons some of the stars used are different for the two groups of plates, and the longer the interval the weaker the connection between the star-places.

From a discussion of the *Right Ascensions* the following results have been derived :—

	Solar Parallax.	*Theoretical Weight.*
Astrographic photographs, Reference Stars,	$8''.793 \pm ''.005$	39·7
,, ,, Comparison Stars,	$8''.809 \pm ''.0052$	39·7
Thompson photographs, Comparison Stars,	$8''.800 \pm ''.0063$	31·9
Combined discussion of Astrographic and Thompson photographs, Comparison Stars,	$8''.800 \pm ''.0044$	71·1

The unit of weight used here is the weight of a comparison between one morning and one evening photograph under practically the most favourable conditions, viz. when Eros undergoes a parallactic displacement of 25″ between the two plates.

The difference between the results derived from the Reference and Comparison Stars is of a systematic character. It is found that if plates taken at a large hour-angle are compared with those taken near the meridian (on the same night and with the telescope on the same side of the pier), the comparison stars are displaced relatively to the reference stars, always in the same direction and in the mean by about ″·07 in R.A. It is not clear whether this displacement depends on the comparison stars being fainter than the reference stars, or their being nearer to the centre of the plate. In either case we must suppose that Eros behaves like the comparison stars rather than like the reference stars. For that reason only the results from the comparison stars are used in obtaining the final value of the parallax. It should be noticed that no diminution of the accidental error would be obtained by taking the mean of the results of the comparison and reference stars; for the two determinations are not independent as regards accidental error, depending, as they do, on the same measures of the image of Eros.

The close agreement between the results derived from the Astrographic and Thompson photographs (Comparison stars), and the fact that the same star-places were used in the two cases, seemed to justify a combined discussion of their results, regarding the two sets of places as homogeneous. There was a considerable gain in thus treating them together, as the two instruments had supplemented one another considerably. In this discussion, comparisons extending over more than a day were rigorously excluded. The result given above must be regarded as the principal conclusion of the whole investigation. The results of the separate comparisons are given in Table I., at the end of the paper.

Without much loss of material, the discussion could be rearranged, so that comparisons were only made between the evening and following morning observations. A very considerable reduction in the probable error (inferred from the discordances) took place (notwithstanding the loss of material), owing to the elimination of erroneous star-places. A comparison was also made between the morning and following evening observations, the probable error in this case being more than twice as great.

Thompson and Astrographic Combined.		*Theoretical Weight.*
Evening to Morning comparisons only,	8″·807 ± ″·0036	58·5
Morning to Evening comparisons only,	8·801 ± ·0080	35·4

The probable error for unit weight of an evening to morning comparison is ± ″·028, and for a morning to evening comparison is ± ″·048. The greater part of this difference in the respective probable errors must be attributed to the errors of star-places.

In a former paper, "On the errors of a photographed réseau" (*Monthly Notices*, vol. lxvii. p. 175), it has been shown that the division errors of the central lines of the réseau between which Eros falls affect systematically the deduced value of the solar parallax. Details were there given of the determination of these errors. The corrections there deduced (additional to those which had been provisionally employed), amounting in the mean to + ″·065 for Astrographic photographs, were applied to the positions of Eros before deducing the preceding results. This quantity was very closely confirmed by a discussion of the residuals of the comparison stars, which indicated + ″·068. The probable errors of the two results separately are, however, about ± ″·006 and ± ″·008 respectively. In order that the effect of the uncertainty in the determination of division errors may be estimated, the results obtained before these corrections were applied are given below.

Provisional Results uncorrected for Residual Division Error.

Astrographic photographs, Reference Stars, .	8″·758 ± ″·0054
,, ,, Comparison Stars,	8·774 ± ·0059
Thompson	8·802 ± ·0071

It will be seen that the result from the Thompson photographs is practically unaltered by the application of the new division errors. This is due to the fact that, in the first place, the scale of the Thompson photographs is twice that of the Astrographic, and that the corrections for division error are consequently halved. Further, the accidental variations in the position of Eros relatively to the lines of the réseau were much greater than in the Astrographic photographs, and during part of the opposition the réseau was not reversed when the telescope passed from E to W of the pier. For Astrographic photographs the application of the division correction ″·065 has increased the deduced value of the solar parallax by ″·035.

A discussion was also made using only comparisons between plates taken with the telescope on the same side of the pier, so that the division error of the réseau was eliminated.

Telescope E only.	*Solar Parallax.*	*Theoretical Weight.*
Astrographic reference stars,	8″·748 ± ″·009	12·5
,, comparison ,,	8·783 ± ·009	12·5

The weight of this determination is small, a large part of the material being wasted. It, however, shows very markedly the difference between the reference and comparison stars.

From the *Declinations* of Eros the following values were obtained :—

	Solar Parallax.	*Theoretical Weight.*
Astrographic photographs, reference stars,	$8''\cdot783 \pm ''\cdot018$	7·1
,, ,, comparison ,,	$8''\cdot819 \pm ''\cdot014$	7·1
Thompson photographs ,,	$8''\cdot783 \pm ''\cdot016$	5·5
Mean (comparison stars),	$8''\cdot801 \pm ''\cdot016$	

No special determination of the division errors of the central lines of the réseau was needed for the declinations. The parallax being of the same sign on both sides of the meridian, comparisons were made between plates taken at a large hour-angle E or W and those near the meridian. About the same proportion of low plates were taken with the telescope E and W (réseau direct and reversed) as meridian plates; accordingly the value of the division error of the central lines of the réseau does not affect systematically the deduced parallax. The method employed for deducing the parallax was to divide the observations into groups, each extending over two or three days, and solve each group separately by least squares. The results from the individual groups for Astrographic Comparison Stars are given in Table II.

TABLE I.

Right Ascensions. Astrographic and Thompson combined.

Date.	No. of Plates compared.	Difference of Parallax.	Weight.	Correction to Solar Parallax (adopted value $8''\cdot800$).
1900.		$''$		$''$
Oct. 14–15	4, 1	·10	·3	−·045
,, 20	3, 1	·13	·4	+·117
,, 21	4, 2	·23	2·3	+·007
,, 26	6, 4	·25	4·8	+·006
,, 27	5, 4	·25	4·5	+·038
,, 28–29	2, 5	·26	3·1	−·009
Nov. 8–9	2, 2	·19	1·2	−·012
,, 9–10	3, 3	·27	3·5	+·012
,, 10–11	2, 3	·27	2·8	+·012
,, 13	5, 5	·28	6·3	−·011
,, 14–15	3, 5	·24	3·5	−·067
,, 22–23	4, 5	·28	5·6	+·009
,, 27	5, 4	·15	1·6	−·005
Dec. 6–7	9, 3	·16	1·9	+·054
,, 9–10	3, 1	·25	1·5	+·011

TABLE I.—*continued.*

Date.	No. of Plates compared.	Difference of Parallax.	Weight.	Correction to Solar Parallax (adopted value 8″·800).
1900.		″		″
Dec. 13	6, 3	21	2·8	− ·044
,, 15–16	6, 6	26	6·5	− ·018
,, 19	5, 5	25	5·0	+ ·003
,, 21	2, 6	27	3·5	+ ·033
,, 28–29	3, 3	12	·7	+ ·033
1901.				
Jan. 5	4, 6	12	1·1	·000
,, 8–9	7, 4	18	2·6	− ·082
,, 13–14	6, 2	22	2·4	+ ·030
,, 15	5, 5	20	3·2	+ ·034
,, 17–18	2, 3	16	1·0	(+ ·277)

Mean Correction + ″·000 ± ″·0044.

The weight is given by the formula $\frac{2mn}{m+n}\left(\frac{\delta\pi}{25''}\right)^2$ where m and n are the numbers of plates in the two groups compared, and $\delta\pi$ is the difference of parallax of Eros between the two groups. No plate is used twice over; *i.e.* if an evening plate is compared with plates taken on the preceding morning, it is not also compared with those taken on the succeeding morning.

The discordant result for Jan. 17–18 has been rejected, as there are several unsatisfactory circumstances in the Thompson photographs on which it depends. On four of the five photographs the images of Eros were noted as difficult to measure, and further, different reference stars had to be used for the two days compared, thus introducing the errors of the star-places which depend on visual observations alone.

TABLE II.

Declinations. Astrographic from Comparison Stars.

Date.	Weight.	Correction to Parallax.
1900.		″
Oct. 14–15	·2	+ ·062
,, 20–21	·4	− ·070
,, 26–27	·6	− ·023
Nov. 9–11	·9	+ ·018
,, 13–15	1·0	+ ·049
,, 22–23	·7	− ·082
Dec. 6–7	·6	+ ·047
,, 13	·5	+ ·040
,, 15–17	·5	+ ·128
,, 19–21	·7	+ ·179
,, 26–29	·3	− ·058

TABLE II.—*continued.*

Date.	Weight.	Correction to Parallax.
1901.		″
Jan. 4–5	·4	+ ·060
,, 8–9	·4	− ·040
,, 14–15	·2	− ·100

Mean Correction + ″·019 ± ″·014.

Royal Observatory, Greenwich:
1907 *April* 9.

The Perturbations of Halley's Comet. By P. H. Cowell and A. C. D. Crommelin.

The differential equations of variation of the four elements, n, e, ϖ, ϵ, the mean motion, eccentricity, longitude of perihelion and epoch are—

$$\frac{1}{m'}\frac{dn}{n} = n_x(a^2\mathrm{X}du) + n_y(a^2\mathrm{Y}du)$$

$$\frac{1}{m'}\,de = e_x(a^2\mathrm{X}du) + e_y(a^2\mathrm{Y}du)$$

$$\frac{1}{m'}\,ed\varpi = \varpi_x(a^2\mathrm{X}du) + \varpi_y(a^2\mathrm{Y}du)$$

$$\frac{1}{m'}\left[d\epsilon - d\varpi\{1 - \sqrt{(1-e^2)}\}\right] = \epsilon_x(a^2\mathrm{X}du) + \epsilon_y(a^2\mathrm{Y}du)$$

where

$$n_x = 3\sin u \qquad n_y = -3\sqrt{(1-e^2)}\cos u$$

$$e_x = -\sqrt{(1-e^2)}\sin u\cos u \qquad e_y = \sqrt{(1-e^2)}\{1 - 2e\cos u + \cos^2 u\}$$

$$\varpi_x = -\sqrt{(1-e^2)}\{1 - e\cos u + \sin^2 u\} \qquad \varpi_y = \sin u\cos u - e\sin u$$

$$\epsilon_x = 2\{-\cos u + e(1+\cos^2 u) - e^2\cos u\} \qquad \epsilon_y = 2\sqrt{(1-e^2)}\{-\sin u + e\sin u\cos u\}$$

where $m'\mathrm{X}$, $m'\mathrm{Y}$ are the components of the disturbing force, the axes of x, y being drawn in the plane of the orbit towards perihelion and parallel to the minor axis respectively.

The quantities n_x, etc., are functions of the eccentricity e, and the eccentric anomaly u only. They are tabulated in Table I. for every odd degree of u, and for the value $\sqrt{(1-e^2)} = ·254000$ or $e = ·967204$. It is clearly unnecessary to tabulate beyond $u = 180°$.

All quantities in the above equations $\frac{dn}{n}$, etc., n_x, etc., $a^2\mathrm{X}du$, etc., are pure numbers of zero dimensions in space and time. All reference to such arbitrary units as the Earth's mean distance and the mean solar day is thereby avoided.

The first part of Table I. gives the comet's x and y co-ordinates divided by the semi-major axis, and also nt, the comet's mean anomaly in circular measure.

TABLE I.

First and Second Quadrants. Unit = 0·00001.

u	$\frac{x}{a}$	$\frac{y}{a}$	nt	$2\pi - nt$
1°	+ 32 64	+ 4 43	0 57	6282 61
3	31 43	13 29	1 74	6281 44
5	28 99	22 14	2 97	6280 22
7	25 34	30 96	4 30	6278 88
9	20 48	39 73	5 78	6277 41
11	14 42	48 46	7 44	6275 75
13	+ 7 17	57 14	9 32	6273 87
15	− 1 28	65 74	11 47	6271 72
17	10 90	74 26	13 92	6269 26
19	21 68	82 69	16 72	6266 46
21	33 62	91 02	19 90	6263 28
23	46 70	99 25	23 51	6259 68
25	60 90	107 34	27 57	6255 61
27	76 20	115 31	32 14	6251 05
29	92 58	123 14	37 24	6245 95
31	110 04	130 82	42 90	6240 28
33	128 53	138 34	49 18	6234 00
35	148 05	145 69	56 10	6227 08
37	168 57	152 86	63 69	6219 49
39	190 06	159 85	72 00	6211 19
41	212 49	166 64	81 04	6202 14
43	235 85	173 23	90 86	6192 32
45	260 10	179 60	101 48	6181 70
47	285 21	185 76	112 94	6170 25
49	311 14	191 70	125 25	6157 93
51	337 88	197 40	138 46	6144 73
53	365 39	202 85	152 58	6130 60
55	393 63	208 06	167 64	6115 54
57	422 56	213 02	183 67	6099 51
59	452 17	217 72	200 69	6082 50
61	482 39	222 15	218 72	6064 47
63	513 21	226 32	237 77	6045 41
65	544 59	230 20	257 88	6025 31
67	576 47	233 81	279 06	6004 13
69	608 84	237 13	301 31	5981 87
71	641 64	240 16	324 68	5958 51
73	674 83	242 90	349 15	5934 04
75	708 38	245 34	374 75	5908 44
77	742 25	247 49	401 49	5881 70
79	776 40	249 33	429 38	5853 81
81	810 77	250 87	458 42	5824 76
83	845 34	252 11	488 63	5794 56
85	880 05	253 03	520 01	5763 18
87	914 87	253 65	552 56	5730 63
89	−949 75	+253 96	586 29	5696 90

u	$\frac{x}{a}$	$\frac{y}{a}$	nt	$2\pi - nt$
91°	− 984 66	+253 96	621 19	5661 99
93	1019 54	253 65	657 28	5625 91
95	1054 36	253 03	694 54	5588 64
97	1089 07	252 11	732 97	5550 21
99	1123 64	250 87	772 58	5510 60
101	1158 01	249 33	813 35	5469 84
103	1192 16	247 49	855 27	5427 91
105	1226 02	245 34	898 35	5384 84
107	1259 58	242 90	942 56	5340 62
109	1292 77	240 16	987 90	5295 28
111	1325 57	237 13	1034 35	5248 83
113	1357 94	233 81	1081 91	5201 28
115	1389 82	230 20	1130 54	5152 64
117	1421 19	226 32	1180 25	5102 94
119	1452 01	222 15	1231 01	5052 18
121	1482 24	217 72	1282 79	5000 39
123	1511 84	213 02	1335 59	4947 60
125	1540 78	208 06	1389 38	4893 81
127	1569 02	202 85	1444 12	4839 06
129	1596 52	197 40	1499 82	4783 37
131	1623 26	191 70	1556 42	4726 76
133	1649 20	185 76	1613 92	4669 26
135	1674 31	179 60	1672 28	4610 91
137	1698 56	173 23	1731 47	4551 72
139	1721 91	166 64	1791 46	4491 72
141	1744 35	159 85	1852 23	4430 95
143	1765 84	152 86	1913 74	4369 44
145	1786 36	145 69	1975 96	4307 22
147	1805 88	138 34	2038 86	4244 33
149	1824 37	130 82	2102 39	4180 79
151	1841 82	123 14	2166 54	4116 65
153	1858 21	115 31	2231 25	4051 93
155	1873 51	107 34	2296 50	3986 68
157	1887 71	99 25	2362 25	3920 94
159	1900 78	91 02	2428 46	3854 73
161	1912 72	82 69	2495 09	3788 10
163	1923 51	74 26	2562 10	3721 08
165	1933 13	65 74	2629 46	3653 72
167	1941 57	57 14	2697 13	3586 06
169	1948 83	48 46	2765 06	3518 13
171	1954 89	39 73	2833 21	3449 98
173	1959 75	30 96	2901 55	3381 64
175	1963 40	22 14	2970 03	3313 16
177	1965 83	13 29	3038 61	3244 57
179	−1967 05	+4 42	3107 26	3175 93

TABLE I.—*continued.*

First Quadrant. Unit = 0·00001.

u	n_x	e_x	ϖ_x	ϵ_x	n_y	e_y	ϖ_y	ϵ_y
1°	+ 52 36	− 1 13	− 8 44	− 2 15	− 761 88	+ 16 66	+ 0 57	− 0 29
3	157 01	3 37	9 36	2 14	760 96	16 64	1 64	0 91
5	261 47	5 60	11 19	2 12	759 10	16 60	2 53	1 62
7	365 61	7 80	13 93	2 03	756 32	16 55	3 09	2 48
9	469 30	9 97	17 57	1 83	752 62	16 49	3 20	3 55
11	572 43	12 08	22 09	1 46	748 00	16 44	2 75	4 90
13	674 85	14 14	27 48 −	0 83	742 47	16 40	+ 1 61	6 58
15	776 46	16 13	33 72 +	0 17	736 04	16 39	− 0 33	8 64
17	877 12	18 04	40 78	1 64	728 70	16 42	3 19	11 15
19	976 70	19 86	48 64	3 71	720 48	16 51	7 06	14 14
21	1075 10	21 58	57 27	6 52	711 39	16 67	12 05	17 67
23	1172 19	23 20	66 64	10 24	701 42	16 94	18 25	21 77
25	1267 86	24 71	76 71	15 03	690 61	17 33	25 74	26 50
27	1361 97	26 10	87 46	21 06	678 95	17 86	34 59	31 88
29	1454 43	27 36	98 83	28 53	666 46	18 56	44 89	37 94
31	1545 11	28 48	110 80	37 62	653 16	19 46	56 67	44 72
33	1633 92	29 47	123 31	48 54	639 07	20 58	70 00	52 25
35	1720 73	30 31	136 32	61 50	624 19	21 96	84 92	60 52
37	1805 44	31 01	149 79	76 72	608 56	23 60	101 45	69 57
39	1887 96	31 55	163 67	94 40	592 18	25 56	119 61	79 39
41	1968 18	31 94	177 92	114 76	575 09	27 86	139 41	90 00
43	2046 00	32 18	192 47	138 03	557 29	30 52	160 85	101 38
45	2121 32	32 26	207 28	164 42	538 82	33 57	183 92	113 54
47	2194 06	32 18	222 31	194 15	519 68	37 05	208 59	126 46
49	2264 13	31 94	237 50	227 42	499 92	40 98	234 82	140 11
51	2331 44	31 55	252 80	264 44	479 54	45 38	262 58	154 49
53	2395 91	31 01	268 16	305 41	458 58	50 30	291 81	169 55
55	2457 46	30 31	283 53	350 51	437 06	55 74	322 44	185 28
57	2516 01	29 47	298 86	399 94	415 02	61 74	354 39	201 61
59	2571 50	28 48	314 09	453 84	392 46	68 32	387 58	218 53
61	2623 86	27 36	329 20	512 39	369 42	75 49	421 91	235 97
63	2673 02	26 10	344 12	575 72	345 94	83 29	457 28	253 88
65	2718 92	24 71	358 81	643 96	322 04	91 72	493 56	272 21
67	2761 52	23 20	373 23	717 23	297 74	100 80	530 65	290 90
69	2800 74	21 58	387 34	795 61	273 08	110 54	568 40	309 87
71	2836 56	19 86	401 10	879 18	248 08	120 96	606 68	329 07
73	2868 91	18 04	414 46	968 00	222 79	132 06	645 35	348 43
75	2897 78	16 13	427 40	1062 11	197 22	143 85	684 25	367 86
77	2923 11	14 14	439 88	1161 52	171 41	156 33	723 23	387 28
79	2944 88	12 08	451 88	1266 22	145 40	169 47	762 13	406 64
81	2963 06	9 97	463 35	1376 19	119 20	183 35	800 79	425 83
83	2977 64	7 80	474 29	1491 39	92 86	197 89	839 03	444 78
85	2988 58	5 60	484 66	1611 72	66 41	213 11	876 70	463 41
87	2995 89	3 37	494 45	1737 12	39 88	228 98	913 61	481 62
89	+2999 54	− 1 13	− 503 64	+1867 44	− 13 30	+245 50	− 949 61	− 499 35

TABLE I.—*continued.*

Second Quadrant. Unit = 0·00001.

u	n_x	e_x	ϖ_x	ϵ_x	n_y	e_y	ϖ_y	ϵ_y
91°	+2999 54	+1 13	−512 21	+2002 55	+ 13 30	+262 65	−984 51	−516 50
93	2995 89	3 37	520 16	2142 30	39 88	280 41	1018 14	532 98
95	2988 58	5 60	527 48	2286 48	66 41	298 75	1050 35	548 73
97	2977 64	7 80	534 17	2434 89	92 86	317 65	1080 96	563 65
99	2963 06	9 97	540 22	2587 30	119 20	337 08	1109 80	577 66
101	2944 88	12 08	545 63	2743 45	145 40	357 00	1136 74	590 70
103	2923 11	14 14	550 41	2903 07	171 41	377 38	1161 60	602 68
105	2897 78	16 13	554 57	3065 87	197 22	398 18	1184 25	613 52
107	2868 91	18 04	558 12	3231 52	222 79	419 37	1204 54	623 18
109	2836 56	19 86	561 06	3399 71	248 08	440 89	1222 34	631 57
111	2800 74	21 58	563 42	3570 07	273 08	462 70	1237 53	638 64
113	2761 52	23 20	565 21	3742 24	297 74	484 76	1249 99	644 34
115	2718 92	24 71	566 46	3915 85	322 04	507 02	1259 61	648 60
117	2673 02	26 10	567 18	4090 49	345 94	529 42	1266 29	651 38
119	2623 86	27 36	567 40	4265 75	369 42	551 91	1269 96	652 65
121	2571 50	28 48	567 15	4441 23	392 46	574 44	1270 53	652 36
123	2516 01	29 47	566 46	4616 50	415 02	596 95	1267 94	650 48
125	2457 46	30 31	565 35	4791 10	437 06	619 38	1262 13	646 98
127	2395 91	31 01	563 85	4964 62	458 58	641 69	1253 07	641 86
129	2331 44	31 55	562 01	5136 60	479 54	663 80	1240 73	635 09
131	2264 13	31 94	559 85	5306 59	499 92	685 67	1225 09	626 67
133	2194 06	32 18	557 41	5474 14	519 68	707 23	1206 15	616 60
135	2121 32	32 26	554 72	5638 80	538 82	728 43	1183 92	604 88
137	2046 00	32 18	551 81	5800 13	557 29	749 20	1158 41	591 53
139	1968 18	31 94	548 74	5957 68	575 09	769 49	1129 68	576 56
141	1887 96	31 55	545 52	6111 01	592 18	789 25	1097 76	560 00
143	1805 44	31 01	542 20	6259 70	608 56	808 41	1062 71	541 88
145	1720 73	30 31	538 80	6403 33	624 19	826 92	1024 61	522 23
147	1633 92	29 47	535 38	6541 48	639 07	844 73	983 55	501 11
149	1545 11	28 48	531 96	6673 75	653 16	861 78	939 62	478 55
151	1454 43	27 36	528 57	6799 78	666 46	878 04	892 93	454 62
153	1361 97	26 10	525 24	6919 18	678 95	893 44	843 61	429 38
155	1267 86	24 71	522 02	7031 61	690 61	907 94	791 78	402 88
157	1172 19	23 20	518 92	7136 73	701 42	921 50	737 59	375 21
159	1075 10	21 58	515 97	7234 24	711 39	934 08	681 18	346 44
161	976 70	19 86	513 21	7323 85	720 48	945 65	622 72	316 64
163	877 12	18 04	510 65	7405 28	728 70	956 16	562 38	285 90
165	776 46	16 13	508 31	7478 30	736 04	965 58	500 33	254 32
167	674 85	14 14	506 23	7542 68	742 47	973 89	436 76	221 97
169	572 43	12 08	504 40	7598 24	748 00	981 06	371 85	188 96
171	469 30	9 97	502 86	7644 79	752 62	987 08	305 81	155 38
173	365 61	7 80	501 61	7682 20	756 32	991 91	238 83	121 34
175	261 47	5 60	500 66	7710 36	759 10	995 54	171 12	86 94
177	157 01	3 37	500 03	7729 18	760 96	997 97	102 88	52 27
179	+ 052 36	+1 13	−499 71	+7738 60	+ 761 88	+999 19	− 34 33	− 17 44

TABLE I.—*continued.*

Second Quadrant. Unit = 0·0001.

u	$\left(\frac{a}{r}\right)^3$	$\frac{3a^3x^2}{r^5}$	$\frac{3a^3xy}{r^5}$	$\frac{3a^3y^2}{r^5}$	u	$\left(\frac{a}{r}\right)^3$	$\frac{3a^3x^2}{r^5}$	$\frac{3a^3xy}{r^5}$	$\frac{3a^3y^2}{r^5}$
91°	95 10	267 50	−68 99	1779	137°	20 09	59 65	6 09	62
93	86 23	243 60	60 60	1507	139	19 31	57 39	5 55	54
95	78 45	222 54	53 42	1282	141	18 61	55 36	5 08	47
97	71 57	203 77	47 16	1092	143	17 96	53 48	4 63	40
99	65 53	187 27	41 80	933	145	17 36	51 73	4 22	34
101	60 16	172 48	37 14	800	147	16 83	50 19	3 85	29
103	55 40	159 32	33 08	687	149	16 34	48 77	3 50	25
105	51 16	147 58	29 53	591	151	15 90	47 48	3 17	21
107	47 36	136 96	26 41	509	153	15 49	46 28	2 87	18
109	43 98	127 54	23 69	440	155	15 13	45 24	2 59	15
111	40 95	119 05	21 30	381	157	14 80	44 28	2 33	12
113	38 22	111 36	19 17	330	159	14 51	43 43	2 08	10
115	35 76	104 40	17 29	286	161	14 25	42 66	1 85	8
117	33 56	98 20	15 64	249	163	14 02	42 00	1 62	6
119	31 55	92 49	14 15	216	165	13 82	41 41	1 41	5
121	29 74	87 34	12 83	188	167	13 64	40 88	1 20	4
123	28 10	82 67	11 64	164	169	13 50	40 48	1 01	3
125	26 61	78 40	10 59	143	171	13 37	40 09	0 81	2
127	25 26	74 54	9 63	125	173	13 28	39 83	0 63	1
129	24 02	70 97	8 77	109	175	13 21	39 63	0 45	1
131	22 90	67 75	8 00	95	177	13 16	39 48	0 27	0
133	21 88	64 83	7 30	82	179	13 14	39 42	− 0 9	0
135	20 95	62 15	6 67	72					

When the disturbing force is due to a planet whose mass is m' times that of the Sun, X and Y are the differentials of

$$V = \frac{1}{\rho} - \frac{xx' + yy' + zz'}{r'^3}$$

where ρ is the distance between planet and comet, x', y', z', r', the heliocentric coordinates and distance of the planet.

We may write $V = V_1 + V_2$

or $V = V_3 + V_4$

where $V_1 = \frac{1}{\rho}$ $V_2 = -\frac{xx' + yy' + zz'}{r'^3}$

$$V_3 = \frac{1}{r} + (xx' + yy' + zz')\left(\frac{1}{r^3} - \frac{1}{r'^3}\right)$$

$$V_4 = \frac{1}{\rho} - \frac{1}{r} - \frac{xx' + yy' + zz'}{r^3}$$

The advantage of the latter form is that V_3 can be dealt with in finite terms, whereas the disturbing forces arising from V_4 are of order $\frac{r'^2}{r^4}$, where r is large. The form $V = V_3 + V_4$ is therefore convenient for the parts of the orbit lying considerably beyond the disturbing planet.

The integrals arising from V_3 are

$$\frac{1}{m'}\frac{1}{n}\int dn = A_1 + \alpha_1'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_1\frac{x'}{a} + \beta_1'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_1\frac{y'}{a}$$

$$\frac{1}{m'}\int de = A_2 + \alpha_2'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_2\frac{x'}{a} + \beta_2'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_2\frac{y'}{a}$$

$$\frac{1}{m'}\,e\int d\varpi = A_3 + \alpha_3'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_3\frac{x'}{a} + \beta_3'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_3\frac{y'}{a}$$

$d\epsilon$ is not integrable, but

$$\frac{1}{m'}\left[\int(2\pi - nt)\frac{dn}{n} + \int d\epsilon - \{1 - \sqrt{(1-e^2)}\}\int d\varpi - \frac{2\pi - nt}{n}\int dn\right]$$

$$= A_4 + \alpha_4'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_4\frac{x'}{a} + \beta_4'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_4\frac{y'}{a}.$$

It is convenient for brevity to call the left-hand side of this equation

$$\frac{1}{m'}\left[\int d\zeta + \sqrt{(1-e^2)}\int d\varpi - \frac{2\pi - nt}{n}\int dn\right]$$

so that $\int d\zeta$ is the perturbation of the mean anomaly at the next perihelion passage.

[The effect in arc of the elementary disturbance dn after an interval $\frac{2\pi}{n} - t$ is $(2\pi - nt)\frac{dn}{n}$.]

In the above formulæ

$$A_1 = -\frac{3a}{r} \qquad A_2 = h\frac{dy}{dt} \qquad A_3 = -h\frac{dx}{dt} \qquad A_4 = -u + \pi$$

$$\alpha_1' = -\frac{3}{na}\frac{dx}{dt} \qquad \alpha_1 = \frac{3}{n^2a}\frac{d^2x}{dt^2} \qquad \beta_1' = -\frac{3}{na}\frac{dy}{dt} \qquad \beta_1 = \frac{3}{n^2a}\frac{d^2y}{dt^2}$$

$$\alpha_2' = -\frac{y}{na^2}\frac{dy}{dt} \qquad \beta_2' = \frac{1}{na^2}\left(2x\frac{dy}{dt} - y\frac{dx}{dt}\right)$$

$$\alpha_2 = \frac{1}{n^2a^2}\left\{ y\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^2 \right\} \qquad \beta_2 = \frac{1}{n^2a^2}\left\{ -x\frac{d^2y}{dt^2} - \frac{dy}{dt}\cdot\frac{dx}{dt} \right\}$$

$$\alpha_3' = \frac{1}{na^2}\left\{ 2y\frac{dx}{dt} - x\frac{dy}{dt} \right\} \qquad \beta_3' = -\frac{x}{na^2}\frac{dx}{dt}$$

$$\alpha_3 = \beta_2 \qquad \beta_3 = \frac{1}{n^2a^2}\left\{ x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 \right\}$$

$$\alpha_4' = -\frac{2x}{a} \qquad \alpha_4 = -\frac{1}{na}\frac{dx}{dt} \qquad \beta_4' = -\frac{2y}{a} \qquad \beta_4 = -\frac{1}{na}\frac{dy}{dt}$$

Arbitrary constants may, of course, be added to the A's. If, however, any constant be added to A_1, we shall have a term proportional to the time in A_4. In A_4 the constant $+\pi$ was introduced to create symmetry about aphelion. It is clearly convenient to introduce the $V = V_3 + V_4$ method, and to drop it at points equidistant from aphelion.

h is twice the area conserved $= na^2\sqrt{(1-e^2)}$.

In this paper the perturbations by Jupiter between the returns of 1835 and 1910 are considered.

A preliminary calculation having shown that the comet will return about 1910 May, the disturbing forces due to Jupiter are calculated on the assumption that the comet moves in an undisturbed ellipse for which $\sqrt{(1-e^2)} = \cdot 254000$, and with a mean motion less than Jupiter's in the ratio $1 : 2\pi \times \cdot 999151$.

The disturbing forces are calculated for every odd degree of the comet's eccentric anomaly. Jupiter is supposed to move in an undisturbed ellipse, and the changes of Jupiter's mean anomaly are $360^\circ \times \cdot 999151$ times the changes of nt as given in Table I.

Jupiter's mean anomaly when $u = 0$ is assumed to have been $79^\circ \cdot 327$.

Jupiter's eccentricity is taken as 0·0482538. The ratio of mean motions implies a ratio of mean distances 0·2939437, for $(2\pi \times \cdot 999151)^2\ (\cdot 2939437)^3 = 1 + \frac{1}{1047 \cdot 35}$.

The relative position of the two ellipses may be described as follows:—

The comet's ascending node on Jupiter's orbit is $46^\circ\ 35'$ in advance of Jupiter's perihelion. The motion of the comet is retrograde and its inclination to Jupiter's orbit is $180^\circ - 18^\circ\ 45'$. The comet's perihelion is $113^\circ\ 48'$ in the direction of its own motion from its node on Jupiter's orbit.

If ξ', η' be Jupiter's coordinates referred to its own axes,

$$\frac{\xi'}{a'} = -\frac{3e'}{2} + \left(1 - \frac{3}{8}e'^2\right)\cos g' + \left(\frac{1}{2}e' - \frac{1}{3}e'^3\right)\cos 2g' + \frac{3}{8}e'^2\cos 3g' + \frac{1}{3}e'^3\cos 4g'$$

$$\frac{y'}{a'} = \left(1 - \frac{5}{8}e'^2\right)\sin g' + \left(\frac{1}{2}e' - \frac{5}{12}e'^3\right)\sin 2g' + \frac{3}{8}e'^2\sin 3g' + \frac{1}{3}e'^3\sin 4g'$$

Expressions for

$$\frac{a'^2\xi'}{r'^3}, \quad \frac{a'^2\eta'}{r'^3}$$

can be derived by a double differentiation.

Resolving parallel to the comet's axes

$$\frac{x'}{a} = +\cdot1034\ 614\ \frac{\xi'}{a'} - \cdot2611\ 992\ \frac{y'}{a'}$$

$$\frac{y'}{a} = -\cdot2664\ 364\ \frac{\xi'}{a'} - \cdot1181\ 556\ \frac{\eta'}{a'}$$

$$\frac{z'}{a} = -\cdot0686\ 316\ \frac{\xi'}{a'} + \cdot0649\ 395\ \frac{\eta'}{a'}$$

and

$$\frac{a^2x'}{r'^3}du = +\cdot1421\ 979\ \frac{a'^2\xi'}{r'^3} - \cdot3589\ 935\ \frac{a'^2\eta'}{r'^3}$$

$$\frac{a^2y'}{r'^3}du = -\cdot3661\ 915\ \frac{a'^2\xi'}{r'^3} - \cdot1623\ 937\ \frac{a'^2\eta'}{r'^3}$$

$$\frac{a^2z'}{r'^3}du = -\cdot0943\ 276\ \frac{a'^2\xi'}{r'^3} + \cdot0892\ 532\ \frac{a'^2\eta'}{r'^3}$$

where $du = 2^\circ$ the interval between successive values of u.

Table II. gives these quantities with argument g' the mean anomaly of Jupiter.

The table is strictly appropriate to the revolution 1835–1910 only; but small linear corrections would make it available for any other revolution: moreover, the small corrections need not be applied except when the distance between Jupiter and the comet is small.

TABLE II.

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
°						
0	+ 98 47	− 253 58	− 65 32	+ 156 98	− 404 25	− 104 13
1	93 67	255 70	64 12	149 32	407 62	102 21
2	88 83	257 73	62 90	141 60	410 83	100 26
3	83 97	259 67	61 65	133 83	413 86	98 26
4	79 07	261 51	60 38	126 01	416 73	96 23
5	74 15	263 27	59 09	118 14	419 43	94 15
6	69 20	264 93	57 78	110 22	421 96	92 04
7	64 22	266 49	56 45	102 27	424 31	89 88
8	59 23	267 97	55 10	94 28	426 50	87 70
9	54 21	269 35	53 73	86 25	428 51	85 49
10	49 17	270 63	52 34	78 20	430 34	83 24
11	44 12	271 82	50 94	70 13	432 00	80 95
12	39 05	272 91	49 51	62 04	433 49	78 65
13	33 97	273 91	48 07	53 93	434 79	76 31
14	28 87	274 81	46 61	45 81	435 93	73 94
15	23 76	275 61	45 14	37 68	436 89	71 55
16	18 65	276 32	43 64	29 56	437 66	69 13
17	13 53	276 93	42 14	21 42	438 27	66 69
18	8 41	277 44	40 62	13 30	438 70	64 23
19	+ 3 28	277 86	39 08	+ 5 19	438 95	61 74
20	− 1 85	278 18	37 53	− 2 91	439 03	59 23
21	6 98	278 40	35 97	10 99	438 93	56 72
22	12 11	278 52	34 39	19 06	438 66	54 17
23	17 23	278 55	32 81	27 09	438 22	51 61
24	22 34	278 48	31 21	35 10	437 61	49 04
25	27 45	278 31	29 60	43 08	436 82	46 46
26	32 55	278 04	27 98	51 02	435 86	43 86
27	37 64	277 68	26 35	58 92	434 75	41 25
28	42 71	277 22	24 71	66 77	433 46	38 63
29	47 77	276 67	23 06	74 59	431 99	36 01
30	52 82	276 02	21 41	82 34	430 38	33 39
31	57 84	275 28	19 74	90 04	428 59	30 74
32	62 84	274 44	18 08	97 70	426 65	28 11
33	67 83	273 50	16 40	105 28	424 54	25 45
34	72 79	272 48	14 72	112 80	422 28	22 81
35	77 72	271 35	13 03	120 26	419 86	20 16
36	82 63	270 14	11 34	127 64	417 29	17 52
37	87 51	268 83	9 65	134 95	414 57	14 88
38	92 36	267 43	7 95	142 19	411 70	12 23
39	97 18	265 94	6 25	149 34	408 70	9 60
40	101 96	264 36	4 54	156 41	405 54	6 97
41	106 72	262 69	2 84	163 41	402 23	4 34
42	111 43	260 93	− 1 14	170 31	398 79	− 1 74
43	116 11	259 08	+ 0 57	177 12	395 22	+ 0 87
44	120 74	257 14	2 28	183 84	391 50	3 47
45	− 125 34	− 255 12	+ 3 98	− 190 46	− 387 67	+ 6 05

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
°						
45	−125 34	−255 12	+ 3 98	−190 46	−387 67	+ 6 05
46	129 89	253 01	5 68	197 00	383 70	8 63
47	134 41	250 82	7 39	203 42	379 60	11 18
48	138 87	248 54	9 08	209 75	375 38	13 72
49	143 29	246 18	10 78	215 98	371 04	16 25
50	147 66	243 74	12 47	222 09	366 58	18 76
51	151 98	241 21	14 16	228 11	362 01	21 26
52	156 26	238 61	15 85	234 02	357 32	23 73
53	160 48	235 92	17 52	239 81	352 54	26 19
54	164 64	233 16	19 20	245 50	347 64	28 62
55	168 75	230 32	20 86	251 07	342 64	31 04
56	172 81	227 40	22 52	256 52	337 54	33 44
57	176 81	224 42	24 17	261 86	332 34	35 81
58	180 75	221 35	25 82	267 09	327 05	38 15
59	184 63	218 22	27 45	272 19	321 67	40 48
60	188 46	215 01	29 08	277 17	316 20	42 77
61	192 22	211 73	30 70	282 05	310 65	45 04
62	195 92	208 38	32 30	286 79	305 01	47 30
63	199 55	204 97	33 90	291 41	299 30	49 51
64	203 12	201 49	35 48	295 91	293 51	51 70
65	206 62	197 94	37 06	300 29	287 65	53 86
66	210 06	194 34	38 62	304 54	281 71	56 00
67	213 43	190 66	40 17	308 67	275 71	58 10
68	216 73	186 93	41 71	312 67	269 66	60 18
69	219 97	183 14	43 23	316 55	263 53	62 21
70	223 13	179 29	44 74	320 32	257 35	64 23
71	226 22	175 38	46 24	323 94	251 11	66 22
72	229 24	171 42	47 72	327 45	244 82	68 16
73	232 19	167 40	49 18	330 83	238 49	70 09
74	235 06	163 33	50 63	334 08	232 10	71 97
75	237 86	159 20	52 07	337 20	225 68	73 81
76	240 59	155 03	53 49	340 21	219 20	75 64
77	243 24	150 81	54 89	343 09	212 70	77 42
78	245 81	146 54	56 27	345 84	206 16	79 18
79	248 31	142 23	57 64	348 46	199 59	80 89
80	250 73	137 87	58 99	350 97	192 98	82 57
81	253 07	133 47	60 32	353 34	186 35	84 22
82	255 33	129 03	61 63	355 60	179 69	85 83
83	257 52	124 55	62 92	357 74	173 01	87 41
84	259 62	120 03	64 19	359 75	166 31	88 95
85	261 65	115 47	65 45	361 63	159 59	90 46
86	263 60	110 88	66 68	363 40	152 85	91 93
87	265 46	106 25	67 89	365 04	146 11	93 37
88	267 24	101 60	69 08	366 57	139 36	94 76
89	268 95	96 91	70 26	367 98	132 59	96 13
90	−270 57	− 92 19	+71 41	−369 27	−125 81	+ 97 45

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
90°	−270 57	− 92 19	+71 41	−360 27	−125 81	+ 97 45
91	272 11	87 44	72 54	370 44	119 04	98 74
92	273 56	82 67	73 64	371 50	112 26	100 00
93	274 94	77 87	74 73	372 42	105 48	101 22
94	276 23	73 05	75 79	373 26	98 71	102 41
95	277 44	68 20	76 83	373 96	91 93	103 55
96	278 57	63 34	77 84	374 55	85 17	104 66
97	279 61	58 46	78 84	375 03	78 42	105 74
98	280 58	53 56	79 81	375 41	71 67	106 77
99	281 45	48 64	80 75	375 66	64 94	107 79
100	282 25	43 71	81 67	375 81	58 22	108 75
101	282 96	38 77	82 57	375 85	51 52	109 68
102	283 59	33 82	83 45	375 79	44 82	110 57
103	284 13	28 85	84 30	375 62	38 17	111 44
104	284 59	23 88	85 12	375 34	31 51	112 26
105	284 97	18 90	85 92	374 96	24 88	113 05
106	285 26	13 91	86 69	374 48	18 28	113 80
107	285 48	8 92	87 44	373 90	11 70	114 52
108	285 61 −	3 93	88 17	373 21 −	5 16	115 21
109	285 65 +	1 06	88 86	372 41 +	1 36	115 85
110	285 62	6 05	89 54	371 53	7 85	116 46
111	285 50	11 04	90 18	370 55	14 31	117 05
112	285 30	16 03	90 81	369 47	20 74	117 59
113	285 02	21 02	91 40	368 29	27 14	118 11
114	284 65	25 99	91 97	367 03	33 49	118 59
115	284 20	30 96	92 51	365 67	39 81	119 03
116	283 68	35 92	93 03	364 21	46 10	119 44
117	283 07	40 88	93 52	362 67	52 35	119 82
118	282 38	45 81	93 98	361 03	58 55	120 15
119	281 62	50 74	94 42	359 31	64 72	120 46
120	280 77	55 65	94 83	357 50	70 84	120 73
121	279 84	60 55	95 21	355 60	76 92	120 98
122	278 84	65 43	95 56	353 63	82 95	121 19
123	277 76	70 29	95 90	351 56	88 94	121 37
124	276 59	75 13	96 20	349 41	94 89	121 52
125	275 35	79 95	96 47	347 18	100 78	121 63
126	274 04	84 75	96 72	344 87	106 64	121 72
127	272 64	89 52	96 94	342 48	112 43	121 77
128	271 18	94 27	97 14	340 02	118 18	121 79
129	269 63	98 99	97 30	337 47	123 88	121 78
130	268 02	103 69	97 44	334 86	129 53	121 75
131	266 32	108 36	97 56	332 16	135 13	121 68
132	264 56	112 99	97 64	329 39	140 67	121 58
133	262 72	117 60	97 71	326 55	146 16	121 42
134	260 80	122 17	97 74	323 64	151 60	121 28
135	−258 82	+126 71	+97 74	−320 65	+156 98	+121 09

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
135°	−258 82	+126 71	+97 74	−320 65	+156 98	+121 09
136	256 77	131 22	97 72	317 60	162 30	120 88
137	254 64	135 68	97 68	314 48	167 56	120 62
138	252 44	140 12	97 60	311 29	172 77	120 35
139	250 18	144 51	97 50	308 03	177 92	120 04
140	247 85	148 86	97 37	304 72	183 01	119 71
141	245 45	153 18	97 22	301 33	188 04	119 35
142	242 99	157 45	97 04	297 88	193 02	118 96
143	240 45	161 68	96 83	294 37	197 94	118 54
144	237 86	165 86	96 60	290 80	202 79	118 09
145	235 20	170 00	96 34	287 17	207 58	117 62
146	232 47	174 10	96 05	283 48	212 30	117 13
147	229 68	178 14	95 74	279 74	216 97	116 61
148	226 83	182 14	95 40	275 93	221 58	116 06
149	223 92	186 09	95 04	272 06	226 12	115 48
150	220 95	189 99	94 65	268 15	230 60	114 87
151	217 92	193 84	94 24	264 19	235 01	114 25
152	214 83	197 64	93 80	260 16	239 36	113 59
153	211 68	201 38	93 33	256 09	243 64	112 91
154	208 48	205 07	92 84	251 96	247 86	112 21
155	205 22	208 71	92 33	247 78	252 00	111 47
156	201 90	212 28	91 79	243 56	256 09	110 72
157	198 53	215 81	91 22	239 27	260 11	109 95
158	195 11	219 27	90 64	234 95	264 06	109 14
159	191 64	222 68	90 02	230 58	267 94	108 32
160	188 11	226 02	89 39	226 16	271 76	107 47
161	184 54	229 31	88 73	221 71	275 51	106 60
162	180 91	232 54	88 04	217 20	279 18	105 70
163	177 24	235 70	87 33	212 64	282 80	104 79
164	173 52	238 80	86 60	208 05	286 33	103 84
165	169 75	241 84	85 85	203 41	289 80	102 88
166	165 94	244 81	85 08	198 74	293 20	101 89
167	162 09	247 72	84 28	194 02	296 53	100 88
168	158 19	250 56	83 46	189 26	299 79	99 85
169	154 25	253 34	82 61	184 47	302 98	98 80
170	150 27	256 04	81 75	179 63	306 10	97 72
171	146 25	258 68	80 86	174 77	309 14	96 63
172	142 19	261 26	79 95	169 86	312 12	95 51
173	138 09	263 76	79 02	164 92	315 01	94 38
174	133 96	266 19	78 07	159 95	317 84	93 21
175	129 79	268 56	77 10	154 93	320 59	92 04
176	125 59	270 85	76 11	149 89	323 27	90 84
177	121 35	273 07	75 10	144 81	325 88	89 61
178	117 08	275 22	74 06	139 71	328 40	88 38
179	112 78	277 29	73 01	134 57	330 87	87 12
180	−108 45	+279 29	+71 94	−129 41	+333 25	+ 85 84

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
180°	−108 45	+279 29	+71 94	−129 41	+333 25	+ 85 84
181	104 10	281 22	70 85	124 21	335 55	84 54
182	99 71	283 07	69 74	118 99	337 78	83 22
183	95 29	284 85	68 62	113 73	339 94	81 89
184	90 86	286 56	67 47	108 45	342 01	80 54
185	86 40	288 18	66 31	103 15	344 01	79 16
186	81 92	289 74	65 13	97 81	345 94	77 77
187	77 41	291 21	63 93	92 46	347 79	76 36
188	72 88	292 61	62 72	87 08	349 56	74 93
189	68 34	293 93	61 49	81 67	351 26	73 49
190	63 78	295 17	60 24	76 25	352 86	72 02
191	59 20	296 34	58 98	70 81	354 40	70 54
192	54 60	297 42	57 70	65 34	355 85	69 03
193	49 99	298 43	56 41	59 84	357 23	67 52
194	45 37	299 35	55 10	54 34	358 52	65 99
195	40 73	300 20	53 77	48 81	359 74	64 44
196	36 08	300 97	52 44	43 27	360 87	62 88
197	31 42	301 66	51 08	37 72	361 92	61 29
198	26 76	302 27	49 72	32 14	362 90	59 70
199	22 09	302 79	48 34	26 53	363 79	58 08
200	17 41	303 24	46 95	20 94	364 60	56 45
201	12 73	303 61	45 54	15 32	365 32	54 80
202	8 04	303 89	44 13	9 69	365 96	53 14
203	− 3 35	304 10	42 70	− 4 05	366 51	51 47
204	+ 1 33	304 22	41 26	+ 1 60	366 99	49 78
205	6 02	304 26	39 81	7 26	367 38	48 07
206	10 71	304 22	38 35	12 94	367 68	46 35
207	15 39	304 10	36 88	18 61	367 90	44 61
208	20 07	303 90	35 40	24 30	368 04	42 87
209	24 75	303 61	33 90	29 99	368 09	41 11
210	29 42	303 25	32 40	35 69	368 04	39 33
211	34 08	302 80	30 90	41 38	367 92	37 54
212	38 73	302 27	29 38	47 11	367 70	35 74
213	43 37	301 66	27 85	52 82	367 41	33 93
214	47 99	300 97	26 32	58 52	367 02	32 09
215	52 61	300 20	24 78	64 23	366 54	30 26
216	57 21	299 34	23 24	69 94	365 97	28 41
217	61 80	298 40	21 68	75 65	365 32	26 54
218	66 37	297 39	20 12	81 36	364 58	24 68
219	70 92	296 29	18 56	87 07	363 74	22 79
220	75 45	295 11	16 99	92 76	362 81	20 89
221	79 96	293 85	15 42	98 45	361 80	18 98
222	84 45	292 51	13 84	104 13	360 69	17 07
223	88 92	291 10	12 26	109 82	359 50	15 14
224	93 36	289 60	10 68	115 48	358 20	13 20
225	+ 97 78	+288 02	+ 9 09	+121 13	+356 82	+ 11 25

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
225°	+ 97 78	+288 02	+ 9 09	+121 13	+356 82	+ 11 25
226	102 16	286 36	7 50	126 78	355 34	9 30
227	106 53	284 63	5 90	132 41	353 78	7 34
228	110 86	282 81	4 31	138 03	352 11	5 36
229	115 16	280 92	2 72	143 64	350 35	3 38
230	119 43	278 95	+ 1 12	149 22	348 51	+ 1 39
231	123 69	276 90	− 0 48	154 79	346 56	− 0 60
232	127 87	274 78	2 07	160 34	344 52	2 61
233	132 04	272 58	3 67	165 86	342 39	4 61
234	136 16	270 30	5 26	171 37	340 16	6 62
235	140 26	267 95	6 86	176 86	337 84	8 65
236	144 31	265 53	8 45	182 31	335 41	10 68
237	148 32	263 03	10 04	187 74	332 90	12 71
238	152 29	260 45	11 62	193 15	330 29	14 75
239	156 22	257 80	13 20	198 52	327 58	16 78
240	160 10	255 08	14 78	203 88	324 78	18 83
241	163 94	252 29	16 36	209 19	321 88	20 88
242	167 74	249 43	17 93	214 47	318 89	22 93
243	171 48	246 50	19 49	219 71	315 79	24 98
244	175 18	243 49	21 05	224 93	312 60	27 04
245	178 82	240 42	22 61	230 09	309 31	29 09
246	182 42	237 28	24 15	235 23	305 93	31 15
247	185 96	234 07	25 69	240 31	302 44	33 21
248	189 45	230 79	27 23	245 37	298 86	35 27
249	192 89	227 45	28 75	250 37	295 19	37 33
250	196 27	224 04	30 27	255 33	291 41	39 38
251	199 59	220 56	31 78	260 23	287 54	41 43
252	202 86	217 03	33 28	265 09	283 58	43 49
253	206 06	213 43	34 77	269 90	279 52	45 54
254	209 21	209 77	36 24	274 66	275 36	47 58
255	212 30	206 04	37 71	279 36	271 10	49 63
256	215 32	202 26	39 17	284 00	266 75	51 66
257	218 28	198 42	40 62	288 58	262 29	53 70
258	221 18	194 52	42 05	293 11	257 76	55 73
259	224 02	190 56	43 47	297 57	253 12	57 74
260	226 78	186 55	44 88	301 97	248 38	59 77
261	229 48	182 48	46 28	306 30	243 56	61 77
262	232 11	178 36	47 66	310 57	238 63	63 77
263	234 68	174 18	49 03	314 77	233 62	65 76
264	237 17	169 96	50 38	318 89	228 51	67 74
265	239 59	165 68	51 72	322 94	223 31	69 71
266	241 94	161 35	53 04	326 92	218 01	71 67
267	244 22	156 98	54 35	330 82	212 64	73 62
268	246 43	152 56	55 64	334 64	207 16	75 56
269	248 56	148 09	56 91	338 38	201 60	77 48
270	+250 61	+143 57	−58 17	+342 03	+195 95	− 79 39

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
270	+250 61	+143 57	−58 17	+342 03	+195 95	− 79 39
271	252 60	139 02	59 41	345 60	190 21	81 29
272	254 50	134 42	60 63	349 09	184 38	83 16
273	256 32	129 78	61 83	352 48	178 47	85 03
274	258 07	125 10	63 02	355 78	172 47	86 87
275	259 74	120 38	64 18	358 99	166 39	88 70
276	261 33	115 63	65 33	362 11	160 23	90 51
277	262 84	110 84	66 45	365 12	153 99	92 31
278	264 27	106 02	67 56	368 04	147 65	94 09
279	265 62	101 16	68 64	370 86	141 25	95 84
280	266 88	96 27	69 70	373 57	134 78	97 57
281	268 06	91 35	70 74	376 18	128 21	99 27
282	269 16	86 41	71 76	378 68	121 58	100 96
283	270 18	81 43	72 76	381 07	114 88	102 62
284	271 11	76 44	73 73	383 35	108 10	104 26
285	271 95	71 41	74 68	385 52	101 26	105 87
286	272 71	66 37	75 61	387 58	94 34	107 45
287	273 39	61 30	76 51	389 51	87 37	109 01
288	273 97	56 22	77 39	391 33	80 32	110 54
289	274 48	51 12	78 25	393 02	73 21	112 04
290	274 89	46 00	79 08	394 60	66 05	113 51
291	275 22	40 86	79 88	396 05	58 83	114 95
292	275 45	35 72	80 66	397 37	51 54	116 36
293	275 60	30 56	81 41	398 57	44 21	117 74
294	275 67	25 39	82 14	399 64	36 83	119 08
295	275 64	20 21	82 84	400 57	29 39	120 38
296	275 52	15 03	83 52	401 37	21 91	121 66
297	275 32	9 84	84 16	402 05	14 40	122 89
298	275 03 +	4 65	84 78	402 57 +	6 83	124 10
299	274 64 −	54	85 38	402 97 −	0 77	125 26
300	274 17	5 73	85 94	403 23	8 42	126 39
301	273 61	10 92	86 48	403 35	16 09	127 48
302	272 96	16 11	86 98	403 33	23 79	128 53
303	272 22	21 29	87 46	403 16	31 52	129 53
304	271 39	26 47	87 92	402 84	39 26	130 50
305	270 47	31 63	88 34	402 39	47 04	131 42
306	269 47	36 79	88 73	401 78	54 84	132 30
307	268 37	41 93	89 10	401 03	62 64	133 13
308	267 18	47 06	89 43	400 14	70 46	133 93
309	265 91	52 17	89 73	399 09	78 29	134 68
310	264 55	57 27	90 01	397 89	86 12	135 38
311	263 10	62 35	90 26	396 54	93 96	136 03
312	261 56	67 40	90 47	395 05	101 80	136 64
313	259 93	72 44	90 65	393 40	109 62	137 20
314	258 22	77 45	90 81	391 60	117 46	137 71
315	+256 42 −	82 43	−90 93	+389 64	−125 25	−138 17

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r^3}du$	$\frac{a^2y'}{r^3}du$	$\frac{a^2z'}{r^3}du$
315°	+256 42	− 82 43	−90 93	+389 64	−125 25	−138 17
316	254 53	87 38	91 02	387 52	133 04	138 59
317	252 56	92 31	91 09	385 26	140 82	138 95
318	250 50	97 21	91 12	382 85	148 57	139 26
319	248 36	102 07	91 12	380 29	156 29	139 52
320	246 13	106 90	91 09	377 57	163 98	139 73
321	243 82	111 69	91 03	374 70	171 64	139 88
322	241 42	116 44	90 93	371 67	179 26	139 99
323	238 95	121 15	90 81	368 49	186 83	140 04
324	236 39	125 83	90 66	365 16	194 37	140 04
325	233 75	130 46	90 47	361 68	201 86	139 98
326	231 03	135 04	90 25	358 04	209 28	139 87
327	228 23	139 58	90 01	354 26	216 66	139 71
328	225 36	144 07	89 73	350 34	223 97	139 49
329	222 40	148 51	89 42	346 26	231 23	139 22
330	219 37	152 90	89 08	342 04	238 40	138 89
331	216 26	157 23	88 71	337 67	245 51	138 51
332	213 08	161 51	88 30	333 15	252 54	138 07
333	209 83	165 74	87 87	328 50	259 49	137 57
334	206 50	169 90	87 41	323 70	266 36	137 02
335	203 10	174 02	86 92	318 76	273 14	136 42
336	199 63	178 06	86 39	313 70	279 83	135 76
337	196 09	182 05	85 84	308 49	286 42	135 05
338	192 48	185 98	85 26	303 14	292 92	134 27
339	188 80	189 83	84 64	297 67	299 31	133 46
340	185 06	193 62	84 00	292 07	305 59	132 57
341	181 25	197 35	83 33	286 33	311 77	131 64
342	177 38	201 00	82 63	280 48	317 84	130 65
343	173 45	204 59	81 90	274 50	323 79	129 61
344	169 46	208 10	81 14	268 40	329 62	128 51
345	165 40	211 54	80 35	262 18	335 33	127 37
346	161 29	214 91	79 54	255 85	340 91	126 16
347	157 13	218 20	78 69	249 41	346 37	124 91
348	152 90	221 41	77 82	242 86	351 69	123 61
349	148 63	224 54	76 92	236 21	356 88	122 25
350	144 30	227 60	76 00	229 44	361 92	120 84
351	139 92	230 58	75 04	222 59	366 83	119 39
352	135 49	233 47	74 06	215 64	371 60	117 88
353	131 01	236 28	73 06	208 59	376 21	116 32
354	126 49	239 01	72 03	201 46	380 68	114 72
355	121 92	241 66	70 97	194 24	385 01	113 07
356	117 31	244 21	69 89	186 93	389 17	111 37
357	112 66	246 69	68 78	179 55	393 18	109 62
358	107 96	249 07	67 65	172 10	397 03	107 84
359	103 24	251 37	66 50	164 58	400 72	106 01
360	+ 98 47	−253 58	−65 32	+156 98	−404 25	−104 13

Table III. now gives, for argument u, the values of g' Jupiter's mean anomaly and $a^2\mathrm{X}du$, $a^2\mathrm{Y}du$, $a^2\mathrm{Z}du$ for the first and fourth quadrants; and $a^2\mathrm{X}_4du$, $a^2\mathrm{Y}_4du$, $a^2\mathrm{Z}_4du$ for the second and third quadrants.

The $\mathrm{V} = \mathrm{V}_3 + \mathrm{V}_4$ method is used for the second and third quadrants. The discontinuity in the numerical values as X, Y, Z are replaced by X_4, Y_4, Z_4, illustrates the value of the method.

The values of Z are not required in this paper, but will be made use of when we compute the change of the plane of the orbit.

We have

$$a^2\mathrm{X}_4 = \left(\frac{a^3}{\rho^3} - \frac{a^3}{r^3}\right)(x' - x) + 3\frac{a^3x^2}{r^5} \cdot \frac{x'}{a} + 3\frac{a^3xy}{r^5} \cdot \frac{y'}{a}$$

$$a^2\mathrm{Y}_4 = \left(\frac{a^3}{\rho^3} - \frac{a^3}{r^3}\right)(y' - y) + 3\frac{a^3xy}{r^5} \quad \frac{x'}{a} + 3\frac{a^3y^2}{r^5} \quad \frac{y'}{a}$$

The computation requires the values of $\frac{a^3}{r^3}$, $\frac{3a^3x^2}{r^5}$, $\frac{3a^3xy}{r^5}$ and $\frac{3a^3y^2}{r^5}$ for the second (and third) quadrants. These functions have therefore been included in Table I.

The third and fourth quadrants are written out in the reverse order, so as to preserve the order of the auxiliary quantities given in Table I.

TABLE III.

First Quadrant. Unit 0·0001.

u	g'	$a^2\mathrm{X}du$	$a^2\mathrm{Y}du$	$a^2\mathrm{Z}du$	u	g'	$a^2\mathrm{X}du$	$a^2\mathrm{Y}du$	$a^2\mathrm{Z}du$
°	°				°	°			
1	79·532	+ 5 56	+ 457	− 2 10	47	119·951	[+ 39 42]	115 57	66 86
3	79·953	6 54	401	2 28	49	124·379	68 44	119 74	80 19
5	80·395	7 34	333	2 41	51	129·130	99 10	105 91	80 82
7	80·874	8 02	2 54	2 50	53	134·209	115 15	78 26	65 00
9	81·406	8 59	1 60	2 54	55	139·626	110 61	51 70	41 81
11	82·003	9 05 +	53	2 52	57	145·392	94 21	35 36	21 78
13	82·679	9 43 −	71	2 47	59	151·514	75 93	28 34	8 43
15	83·453	9 71	2 14	2 35	61	157·999	60 18	26 78 +	66
17	84·334	9 91	3 79	2 18	63	164·852	47 43	27 74 −	3 53
19	85·341	10 01	5 68	1 93	65	172·085	37 07	29 68	5 55
21	86·485	10 03	7 86	1 57	67	179·703	28 35	31 78	6 26
23	87·783	9 93	10 39	1 11	69	187·707	20 65	33 69	6 14
25	89·244	9 74	13 31 −	48	71	196·113	13 52	35 17	5 47
27	90·888	9 40	16 71 +	36	73	204·914	6 73	36 06	4 39
29	92·722	8 93	20 71	1 48	75	214·122 +	11	36 26	2 98
31	94·758	8 32	25 45	2 99	77	223·741 −	6 41	35 60 −	1 30
33	97·017	7 59	31 11	5 03	79	233·773	12 82	33 97 +	60
35	99·506	6 80	37 97	7 85	81	244·218	19 03	31 22	2 67
37	102·236	6 13	46 33	11 76	83	255·084	24 87	27 21	4 87
39	105·225	5 94	56 66	17 27	85	266·372	30 11	21 83	7 11
41	108·477	7 05	69 32	25 03	87	278·080	34 39	15 02	9 28
43	112·009	11 02	80 31	35 85	89	290·212	− 37 27 −	6 83 +	11 24
45	115·829	20 62	92 17	50 22					

TABLE III.—*continued.*

Second Quadrant. Unit 0·0001.								*Third Quadrant.* Unit 0·0001.							
u	g'	a^2X_4du		a^2Y_4du		a^2Z_4du		u	g'	a^2X_4du		a^2Y_4du		a^2Z_4du	
91°	302°·77	+	47	−	21	+	15	269°	315°·91	+	25	+	30	+	14
93	315·75		43	−	8		14	267	302·94		36		22		13
95	329·15		36	+	4		12	265	289·53		41		12		11
97	342·97		23		12		9	263	275·71		39	+	2		8
99	357·22	+	8		16		6	261	261·46		32	−	7		5
101	11·88	−	5		15		3	259	246·80		21		13	+	2
103	26·96		15	+	9		1	257	231·72	+	9		16		0
105	42·46		17		0		0	255	216·22	−	3		15	−	1
107	58·36	−	12	−	10		1	253	200·32		13		10	−	1
109	74·67	+	1		18		5	251	184·01		16	−	3		0
111	91·38		16		19		9	249	167·30		14	+	6	+	2
113	108·48		26	−	13		11	247	150·20	−	5		13		6
115	125·98		26		0		11	245	132·70	+	8		15		8
117	143·86		17	+	9		9	243	114·83		20		11		9
119	162·11	+	5		11		5	241	96·57		23	+	2		8
121	180·74	−	5		8		2	239	77·94		16	−	7		4
123	199·73		9	+	3		0	237	58·95	+	6		9		1
125	219·08		7	−	2		0	235	39·60	−	3		7		0
127	238·77	−	3		5		0	233	19·91		6	−	1		0
129	258·80	+	2		6		1	231	359·88	−	5	+	3		1
131	279·16		6		5		2	229	339·52	+	1		5		2
133	299·84		8	−	2		2	227	318·83		5		4		2
135	320·84		7	+	1		2	225	297·85		8	+	1		2
137	342·13	+	3		3		2	223	276·56		7	−	1		2
139	3·71	−	2		3		1	221	254·97	+	5		3		1
141	25·56		5	+	1		0	219	233·11		0		4		0
143	47·69	−	3	−	3		0	217	210·99	−	3	−	3		0
145	70·07	+	2		5		1	215	188·61		5		0		0
147	92·69		7	−	4		3	213	165·99	−	3	+	3		1
149	115·54		9		0		3	211	143·13	+	2		5		3
151	138·62	+	6	+	3		3	209	120·06		7	+	3		3
153	161·90		0		4		2	207	96·78		8	−	1		3
155	185·37	−	2	+	2		0	205	73·31	+	5		3		1
157	209·02		4	−	1		0	203	49·67	−	1		3		0
159	232·83	−	1		3		0	201	25·85		3	−	1		0
161	256·80	+	1		3		1	199	1·89	−	2	+	2		0
163	280·90		4	−	2		1	197	337·78	+	1		2		1
165	305·13		4		0		1	195	313·55		4	+	1		1
167	329·47	+	2	+	2		1	193	289·21		5	−	1		1
169	353·90	−	1		2		1	191	264·78	+	3		2		1
171	18·42		3	+	1		0	189	240·27		0		3		0
173	43·00	−	2	−	2		0	187	215·68	−	2	−	2		0
175	67·63	+	2		3		1	185	191·05		3	+	1		0
177	92·30		6	−	2		2	183	166·38	−	1		3		1
179	116·99	+	6	+	1	+	3	181	141·69	+	3	+	3	+	2

TABLE III.—*continued.*

Fourth Quadrant. Unit 0·0001.

u	g'	$a^2X\,du$	$a^2Y\,du$	$a^2Z\,du$	u	g'	$a^2X\,du$	$a^2Y\,du$	$a^2Z\,du$
359°	179°·146	− 1 31	− 4 52	− 1 30	313°	138°·731	33 81	10 33	3 72
357	178·726	− 19	5 50	1 80	311	134·300	37 04	14 23	2 96
355	178·287	+ 1 19	6 31	2 21	309	129·552	40 83	18 57	1 99
353	177·805	1 97	6 96	2 59	307	124·469	45 36	23 22	− 82
351	177·276	3 04	7 48	2 92	305	119·052	50 76	27 80	+ 45
349	176·679	4 12	7 88	3 21	303	113·286	56 97	31 61	1 60
347	176·003	5 22	8 18	3 48	301	107·168	63 38	33 76	2 21
345	175·229	6 33	8 37	3 73	299	100·683	68 57	33 57	1 73
343	174·344	7 48	8 47	3 93	297	93·827	70 59	31 38	+ 39
341	173·337	8 66	8 46	4 16	295	86·597	68 39	28 82	− 1 59
339	172·193	9 89	8 36	4 35	293	78·979	62 60	27 71	3 22
337	170·899	11 18	8 14	4 52	291	70·972	54 83	28 75	3 91
335	169·435	12 51	7 81	4 69	289	62·569	46 38	31 51	3 55
333	167·779	13 91	7 35	4 84	287	53·768	37 75	35 12	2 32
331	165·960	15 38	6 75	4 97	285	44·560	28 94	38 75	− 43
329	163·921	16 93	5 97	5 08	283	34·941	19 86	41 72	+ 1 87
327	161·662	18 56	4 98	5 16	281	24·909	10 42	43 45	4 42
325	159·173	20 30	3 83	5 21	279	14·460	+ 74	43 50	7 01
323	156·442	22 13	2 33	5 18	277	3·598	− 8 88	41 52	9 43
321	153·457	24 09	− 58	5 11	275	352·310	18 03	37 36	11 50
319	150·202	26 20	+ 1 52	4 94	273	340·602	26 00	31 08	12 98
317	146·670	28 48	4 01	4 68	271	328·470	− 32 36	+ 22 97	+13 75
315	142·850	31 00	6 93	4 28					

Table IV. gives for argument u

$$\frac{1}{m'}\frac{dn}{n}\quad \frac{1}{m'}de,\ \frac{1}{m'}ed\varpi,\ \frac{1}{m'}\left[d\epsilon - d\varpi\{1-\sqrt{(1-e^2)}\}\right] \text{ and } \frac{1}{m'}\frac{dn}{n}\times(2\pi - nt)$$

The mechanical quadratures for the second and third quadrants require to be supplemented by the definite integrals arising from V_3.

TABLE IV.

First Quadrant.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{d\epsilon}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
°					
1	− ·0319	+ ·0007	− ·0005	− ·0001	− 0·200
3	·0202	·0005	·0005	·0001	0·127
5	− ·0061	+ ·0002	·0007	·0003	− 0·038
7	+ ·0101	− ·0002	·0010	·0003	+ 0·063
9	·0283	·0006	·0014	·0003	0·178
11	·0478	·0010	·0020	·0001	0·300
13	·0689	·0014	·0026	− ·0001	0·432
15	·0912	·0020	·0033	+ ·0002	0·572
17	·1145	·0024	·0039	·0006	0·718
19	·1387	·0029	·0045	·0012	0·869
21	·1637	·0035	·0048	·0021	1·025
23	·1893	·0041	·0047	·0033	1·185
25	·2154	·0047	·0041	·0050	1·347
27	·2415	·0055	− ·0024	·0073	1·510
29	·2679	·0062	+ ·0005	·0104	1·673
31	·2948	·0074	·0052	·0145	1·840
33	·3228	·0086	·0124	·0200	2·012
35	·3540	·0104	·0229	·0272	2·204
37	·3926	·0128	·0378	·0369	2·442
39	·4476	·0164	·0581	·0506	2·780
41	·5375	·0216	·0841	·0705	3·334
43	·6731	·0280	·1080	·0966	4·168
45	·9340	·0376	·1268	·1385	5·774
47	1·4655	·0555	·1535	·2226	9·043
49	2·1482	·0710	·1187	·3234	13·228
51	2·8184	·0794	+ ·0276	·4257	17·318
53	3·1178	·0751	− ·0804	·4844	19·114
55	2·9442	·0623	·1469	·4835	18·005
57	2·5171	·0496	·1563	·4481	15·353
59	2·0637	·0410	·1287	·4065	12·552
61	1·6779	·0367	·0852	·3716	10·176
63	1·3638	·0355	− ·0364	·3435	8·245
65	1·1035	·0364	+ ·0135	·3195	6·649
67	·8775	·0386	·0628	·2957	5·269
69	·6704	·0417	·1115	·2687	4·010
71	·4707	·0452	·1592	·2346	2·805
73	·2734	·0488	·2048	·1907	1·622
75	+ ·0747	·0522	·2476	·1346	+ 0·441
77	− ·1264	·0548	·2857	+ ·0634	− 0·743
79	·3281	·0561	·3168	− ·0242	1·921
81	·5267	·0553	·3382	·1289	3·068
83	·7152	·0520	·3463	·2499	4·144
85	·8854	·0448	·3373	·3842	5·103
87	1·0243	·0332	·3072	·5251	5·870
89	− 1·1170	− ·0164	+ ·2526	− ·6619	− 6·363
Sums,	+ 24·3392	− 1·2575	+ 3·0688	+ 3·5259	+ 150·679

TABLE IV.—*continued.*

Second Quadrant. V_4 only.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{d\epsilon}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}(2\pi-nt)$
91°	+ ·0141	− ·0006	− ·0003	+ ·0105	+ 0·080
93	·0129	− ·0002	·0014	·0096	0·073
95	·0108	+ ·0001	·0023	·0080	0·060
97	·0069	·0004	·0025	·0049	0·038
99	+ ·0026	·0005	·0022	+ ·0012	+ 0·014
101	− ·0013	·0005	·0014	− ·0023	− 0·007
103	·0042	+ ·0003	− ·0002	·0049	0·023
105	·0049	·0000	+ ·0009	·0052	0·026
107	·0036	− ·0004	·0019	− ·0033	0·019
109	− ·0001	·0008	·0021	+ ·0014	− 0·001
111	+ ·0040	·0009	+ ·0015	·0069	+ 0·021
113	·0069	− ·0004	− ·0001	·0104	0·036
115	·0071	+ ·0001	·0015	·0102	0·037
117	·0048	·0005	·0021	·0063	0·024
119	+ ·0017	·0006	·0017	+ ·0014	+ 0·009
121	− ·0010	·0005	− ·0007	− ·0027	− 0·005
123	·0022	+ ·0002	+ ·0001	·0044	0·011
125	·0018	− ·0001	·0007	·0033	0·009
127	− ·0009	·0003	·0008	− ·0012	− 0·004
129	+ ·0002	·0004	·0006	+ ·0014	+ 0·001
131	·0012	·0003	+ ·0003	·0035	0·006
133	·0017	− ·0001	− ·0002	·0045	0·008
135	·0016	+ ·0001	·0005	·0038	0·007
137	+ ·0008	·0002	·0005	+ ·0015	+ 0·004
139	− ·0002	·0002	− ·0002	− ·0014	− 0·001
141	·0008	+ ·0001	+ ·0002	·0032	0·004
143	− ·0007	− ·0002	·0005	− ·0017	− 0·003
145	·0000	·0004	+ ·0004	+ ·0016	0·000
147	+ ·0008	− ·0003	·0000	·0048	+ 0·003
149	·0014	·0000	− ·0005	·0060	0·006
151	·0011	+ ·0003	·0006	+ ·0040	0·005
153	+ ·0003	·0004	·0003	− ·0002	+ 0·001
155	− ·0002	+ ·0002	− ·0001	·0015	− 0·001
157	·0006	− ·0001	+ ·0003	·0029	0·002
159	·0003	·0003	·0003	− ·0006	− 0·001
161	− ·0001	·0003	+ ·0001	+ ·0008	0·000
163	+ ·0003	− ·0002	− ·0001	·0031	+ 0·001
165	·0003	·0000	·0002	·0030	0·001
167	+ ·0002	+ ·0002	− ·0002	+ ·0015	+ 0·001
169	·0000	·0002	·0000	− ·0008	0·000
171	·0000	+ ·0001	+ ·0002	·0023	0·000
173	− ·0003	− ·0002	+ ·0001	− ·0015	− 0·001
175	·0001	·0003	·0000	+ ·0016	0·000
177	− ·0001	− ·0002	− ·0003	·0046	0·000
179	+ ·0001	+ ·0001	− ·0003	+ ·0046	0·000
Sums,	+ ·0584	− ·0012	− ·0094	+ ·0777	+ 0·318

TABLE IV.—*continued.*

Third Quadrant. V_4 only.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{d\epsilon}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
269°	− ·0075	+ ·0008	+ ·0017	+ ·0065	− 0·005
267	·0107	·0006	+ ·0003	·0089	0·007
265	·0122	·0004	− ·0009	·0101	0·008
263	·0116	+ ·0001	·0019	·0096	0·009
261	·0096	− ·0002	·0025	·0079	0·007
259	·0064	·0005	·0026	·0050	0·005
257	− ·0029	·0006	·0024	+ ·0016	− 0·002
255	+ ·0006	·0006	·0016	− ·0018	+ 0·001
253	·0035	·0004	− ·0005	·0048	0·003
251	·0044	− ·0001	+ ·0005	·0056	0·004
249	·0041	+ ·0003	·0015	·0046	0·004
247	+ ·0018	·0006	·0019	− ·0011	+ 0·002
245	− ·0017	·0008	·0014	+ ·0041	− 0·002
243	·0049	+ ·0005	+ ·0003	·0089	0·006
241	·0059	·0000	− ·0010	·0099	0·007
239	·0044	− ·0004	·0018	·0066	0·006
237	− ·0019	·0005	·0014	+ ·0022	− 0·003
235	+ ·0004	·0004	− ·0007	− ·0019	+ 0·001
233	·0014	− ·0001	+ ·0002	·0031	0·002
231	+ ·0013	+ ·0002	·0007	− ·0024	+ 0·002
229	·0000	·0003	·0005	+ ·0008	0·000
227	− ·0009	·0003	+ ·0002	·0029	− 0·001
225	·0016	+ ·0001	− ·0003	·0046	0·003
223	·0015	− ·0001	·0005	·0040	0·003
221	·0012	·0002	·0006	+ ·0028	− 0·002
219	− ·0002	·0003	·0004	− ·0002	0·000
217	+ ·0003	− ·0002	− ·0001	·0021	+ 0·001
215	·0009	·0000	+ ·0003	·0032	0·002
213	+ ·0007	+ ·0003	·0005	− ·0018	+ 0·001
211	·0000	·0004	+ ·0004	+ ·0015	0·000
209	− ·0008	+ ·0003	− ·0001	·0049	− 0·002
207	·0012	− ·0001	·0005	·0055	0·003
205	·0008	·0003	·0005	+ ·0034	− 0·002
203	− ·0001	·0003	− ·0001	− ·0008	0·000
201	+ ·0002	− ·0001	+ ·0001	·0022	0·000
199	+ ·0003	+ ·0002	+ ·0002	− ·0014	+ 0·001
197	·0000	·0002	·0000	+ ·0008	0·000
195	− ·0002	+ ·0001	− ·0001	·0030	− 0·001
193	·0004	− ·0001	·0003	·0038	0·001
191	·0003	·0002	·0003	+ ·0023	0·001
189	·0002	·0003	− ·0001	·0000	− 0·001
187	− ·0001	− ·0002	+ ·0001	− ·0015	0·000
185	+ ·0002	+ ·0001	·0002	·0023	+ 0·001
183	·0002	·0003	+ ·0001	− ·0008	0·001
181	+ ·0002	+ ·0003	− ·0001	+ ·0023	+ 0·001
Sums,	− ·0687	+ ·0010	− ·0102	+ ·0823	− 0·060

TABLE IV.—*continued.*

Fourth Quadrant.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
°					
359	+ ·0351	− ·0008	+ ·0001	0000	0·000
357	·0422	·0009	·0001	− ·0001	0·000
355	·0448	·0009	+ ·0001	·0001	0·000
353	·0454	·0010	− ·0001	·0002	0·000
351	·0420	·0009	·0003	·0004	0·000
349	·0353	·0008	·0007	·0005	0·000
347	·0255	·0006	·0013	·0005	0·000
345	+ ·0125	·0004	·0021	·0007	0·000
343	− ·0039	− ·0001	·0034	·0008	0·000
341	·0236	+ ·0003	·0048	·0009	0·000
339	·0468	·0007	·0067	·0009	− 0·001
337	·0740	·0012	·0090	·0007	0·002
335	·1047	·0017	·0116	− ·0002	0·003
333	·1396	·0023	·0147	+ ·0006	0·004
331	·1787	·0029	·0182	·0018	0·007
329	·2226	·0036	·0222	·0037	0·010
327	·2715	·0045	·0264	·0064	0·013
325	·3254	·0054	·0310	·0102	0·018
323	·3853	·0064	·0355	·0154	0·025
321	·4514	·0075	·0401	·0222	0·033
319	·5244	·0088	·0445	·0287	0·043
317	·6050	·0104	·0483	·0434	0·055
315	·6949	·0123	·0516	·0589	0·071
313	·7955	·0147	·0537	·0787	0·090
311	·9097	·0176	·0546	·1041	0·114
309	1·0410	·0213	·0544	·1367	0·144
307	1·1933	·0258	·0538	·1779	0·182
305	1·3689	·0309	·0543	·2294	0·229
303	1·5646	·0363	·0583	·2915	0·289
301	1·7623	·0412	·0683	·3614	0·354
299	1·9232	·0441	·0841	·4305	0·421
297	1·9955	·0445	·0994	·4861	0·474
295	1·9523	·0433	·1032	·5189	0·503
293	1·8112	·0424	·0866	·5296	0·505
291	1·6141	·0436	− ·0490	·5253	0·486
289	1·3938	·0473	+ ·0052	·5115	0·453
287	1·1612	·0532	·0701	·4878	0·405
285	·9150	·0604	·1414	·4499	0·343
283	·6520	·0680	·2143	·3925	0·262
281	·3701	·0749	·2840	·3086	0·159
279	− ·0738	·0799	·3449	·1954	− 0·034
277	+ ·2258	·0815	·3905	+ ·0523	+ 0·110
275	·5140	·0786	·4149	− ·1175	0·267
273	·7665	·0703	·4125	·3020	0·424
271	+ ·9676	+ ·0560	+ ·3811	− ·4896	+ 0·567
Sums,	− 23·7926	+ 1·1374	+ 1·4670	+ 5·5443	− 4·364

In order to calculate the definite integrals arising from V_3 we require values of Jupiter's coordinates and velocities for $u = 90°$ and $u = 270°$. These are also set down for $u = 180°$ so that the two quadrants may be exhibited separately.

	$u=90°$	$u=180°$	$u=270°$
A_1	−3·000	−1·525	−3·000
α_1'	+3·000	0·000	−3·000
α_1	+2·902	+0·775	+2·902
β_1'	0·000	+0·387	0·000
β_1	−0·762	0·000	+0·762
A_2	0·000	−0·0328	0·000
α_2'	0·000	0·000	0·000
α_2	−0·0645	+0·0167	−0·0645
β_2'	+0·2540	+0·5080	+0·2540
$\alpha_3=\beta_2$	−0·2457	0·000	+0·2457
A_3	+0·2540	0·000	−0·2540
α_3'	−0·5080	−0·2540	−0·5080
β_3'	−0·9672	0·000	+0·9672
β_3	+0·0646	−0·5083	+0·0646
A_4	+1·571	0·000	−1·571
α_4'	+1·934	+3·934	+1·934
α_4	+1·000	0·000	−1·000
β_4'	−0·508	0·000	+0·508
β_4	0·000	+0·129	0·000
$\frac{1}{na}\frac{dx'}{dt}$	−0·0712	+0·5777	−0·8825
$\frac{x'}{a}$	+0·2755	−0·2691	+0·2409
$\frac{1}{na}\frac{dy'}{dt}$	−1·8676	+1·6911	−1·7005
$\frac{y'}{a}$	+0·0127	+0·1007	−0·1177

Hence for the integrals arising from V_3

	$u=90°$	$u=180°$	$u=270°$
$\frac{1}{m'}\cdot\frac{1}{n}\int dn$	−2·424	−1·080	+0·257
$\frac{1}{m'}\int de$	−0·4953	+0·8218	−0·4763
$\frac{1}{m'}e\int d\varpi$	+2·0296	−0·1979	−1·3988
$\frac{1}{m'}\left[\int d\zeta+\sqrt{(1-e^2)}\int d\varpi-\frac{2\pi-nt}{n}\int dn\right]$	+2·658	+2·286	−4·383

Hence for $\frac{1}{m'} \cdot \frac{1}{n}\int dn$,

First quadrant	. . .	+ 24·3392
Second quadrant (V_4)	. . .	+ ·0584
,, (V_3)	. . .	+ 1·344
Third quadrant (V_4)	. . .	− 0·0687
,, (V_3)	. . .	+ 1·337
Fourth quadrant	. . .	− 23·7926
	Sum	+ 3·217

or substituting $m' = \frac{1}{1047{\cdot}35}$

$$\frac{1}{n}\int dn = +{\cdot}003072 \left[\text{de Pontécoulant} = +{\cdot}003441\right].$$

Similarly

$$\int de = m'\left[-1{\cdot}2575 - {\cdot}0012 + 1{\cdot}3171 + {\cdot}0010 - 1{\cdot}2981 + 1{\cdot}1374\right]$$

$$= m' \times -0{\cdot}1013 = -{\cdot}000097$$

$$\int d\varpi = \frac{m'}{e}\left[+3{\cdot}0688 - {\cdot}0094 - 2{\cdot}2275 - {\cdot}0102 - 1{\cdot}2009 + 1{\cdot}4670\right]$$

$$= \frac{m'}{e} \times +1{\cdot}0878 = +{\cdot}001074 \text{ radians} \left[\text{de Pontécoulant} = +{\cdot}001279\right].$$

Lastly, in order to calculate $\int d\zeta$; in extracting the results of the mechanical quadratures we use the formula

$$\int d\zeta = \int (2\pi - nt)\frac{dn}{n} + \left[\int d\epsilon - \left\{1 - \sqrt{(1-e^2)}\right\}\int d\varpi\right] - \frac{\sqrt{(1-e^2)}}{e} e \int d\varpi$$

in extracting the results of the integrations arising from V_3 we use the formula

$$\int d\zeta = \left[\int d\zeta + \sqrt{(1-e^2)}\int d\varpi - \frac{2\pi - nt}{n}\int dn\right] - \frac{\sqrt{(1-e^2)}}{e} e \int d\varpi + (2\pi - nt)\frac{1}{n}\int dn$$

From the mechanical quadratures

$$\int \left(2\pi - nt\right)\frac{dn}{n} = m'\left[+150{\cdot}679 + 0{\cdot}318 - 0{\cdot}060 - 4{\cdot}364\right]$$

$$= m' \times +146{\cdot}573 = +0{\cdot}139947 \text{ radians}$$

Also

$$\int de - 0{\cdot}746 \int d\varpi = m'\left[+3{\cdot}5259 + 0{\cdot}0777 + 0{\cdot}0823 + 5{\cdot}5443\right]$$

$$= m' \times +9{\cdot}2302 = +0{\cdot}008813 \text{ radians.}$$

Hence $\int d\zeta$ (part from mechanical quadratures only)

$= +0{\cdot}139947 \quad +0{\cdot}008813 \quad -0{\cdot}000273$

$= +0{\cdot}148487$ radians.

From the definite integrals we have for $\frac{1}{m'}\int d\zeta$

Integral between limits of $\int d\zeta + \sqrt{(1-e^2)}\int d\varpi - \frac{2\pi - nt}{n}\int dn$ =	− 7·041
$-\frac{\sqrt{(1-e^2)}}{e} \times e\int d\varpi$ taken between limits . . . =	+ 0·900
$+\left(2\pi - nt\right)\frac{1}{n}\int dn$ at the upper limit . . . =	+ 0·155
minus ditto at the lower limit . . . . . =	+ 13·767
Sum	+ 7·781

Hence $\int d\zeta$ (part from integral only) $= +0{\cdot}007429$ radians.

Hence the whole perturbation $\int d\zeta = +{\cdot}15592$ radians.

It should be stated that several approximations have been introduced; in particular the square of the mass of Jupiter has been neglected, or in other words the disturbing forces have been computed as if the comet was in a certain mean undisturbed path.

Our result for $\int de$ is one fifty-fifth part of that given by de Pontécoulant, *C.R.*, lviii. p. 827. This renders it extremely probable that de Pontécoulant's error consisted in the omission of the factor, circular measure of one degree.

De Pontécoulant's value of $\int d\zeta$ is $+{\cdot}15302$ radians; this implies that, considering Jupiter's action only, his value of the next perihelion passage is thirteen days later than ours.

Magnitude of o Ceti (Mira), 1906 December 14–1907 February 16, as observed at the Radcliffe Observatory, Oxford.

(*Communicated by the Radcliffe Observer.*)

Information having been received early in December that the increase in the light of Mira Ceti suggested a maximum of abnormal brightness, observations of this variable were undertaken by Mr Wickham and Mr Robinson.

These observations are made by Argelander's method, and are all naked-eye estimations.

When the observations commenced the variable had already considerably exceeded its brightness on the occasion of the last maximum, but a curve drawn to represent the observed magnitudes does not clearly indicate a maximum, the epoch of which may have occurred previous to December 14. Unfortunately, observations were rendered impracticable on December 11–13 by bad weather which prevailed. The date of maximum derived from Chandler's "Revision of Elements of Third Catalogue of Variable Stars" (*A. J.* 553) is 1906 December 19.*

The variable was so bright on the earlier nights that it was necessary to take comparison-stars situated at considerable distances from it, and at various altitudes, but these are distributed in such a way that the resulting mean will, it is hoped, be fairly free from the effects of atmospheric absorption. The proximity of α Ceti, the colour of which was similar to that of the variable, was found advantageous, but the observers met with the usual difficulty in comparing the dull yellow light of the variable with that of lustrous white stars.

Some notes on colour are included amongst the "Observers' Remarks."

It is worth mentioning that in the *Harvard Annals, Photometry*, vol. xiv. p. 121, α Ceti is described as "O," or pronounced orange, whilst in the recently published vol. xvii. of the Potsdam Astrophysical Observatory this star is characterised as "G—," or between light yellow and yellow.

* It may be well to point out here that the second term of the periodic inequality as given in the footnote to the catalogue is incorrectly printed. The coefficient should be $12^{m}\cdot3$ (not $12^{m}\cdot3$), the unit being a day.

TABLE I.

Stars used for Comparison.

Ref. No.	Name of Star.	R.A. 1900. h m	N.P.D. 1900. ° ′	Adopted Mag.	Ref. No.	Name of Star.	R.A. 1900. h m	N.P.D. 1900. ° ′	Adopted Mag.
1	α Androm.	0 3·2	61 28	2·15	20	α Persei	3 17·2	40 30	1·90
2	γ Pegasi	0 8·1	75 22	2·87	21	ϵ Eridani	3 28·2	99 48	3·82
3	β Ceti	0 38·6	108 32	2·26	22	δ Eridani	3 38·5	100 6	3·72
4	γ Cassiop.	0 50·7	29 49	2·25	23	γ Eridani	3 53·4	103 48	3·27
5	η Ceti	1 3·6	100 43	3·61	24	Aldebaran	4 30·2	73 41	1·06
6	β Androm.	1 4·1	54 55	2·37	25	ι Aurigæ	4 50·5	57 0	2·86
7	θ Ceti	1 19·0	98 42	3·83	26	β Eridani	5 2·9	95 13	2·87
8	η Piscium	1 26·1	75 10	3·71	27	Rigel	5 9·7	98 19	0·34
9	β Arietis	1 49·1	69 41	2·73	28	γ Orionis	5 19·8	83 44	1·70
10	α Piscium	1 56·9	87 43	3·94	29	β Tauri	5 20·0	61 29	1·78
11	γ Androm.	1 57·8	48 9	2·20	30	δ Orionis	5 26·9	90 22	2·48
12	α Arietis	2 1·5	67 1	2·23	31	ϵ Orionis	5 31·1	91 16	1·68
13	ξ^2 Ceti	2 22·8	81 59	4·34	32	ζ Orionis	5 35·7	92 0	1·89
14	δ Ceti	2 34·4	90 6	4·00	33	κ Orionis	5 43·0	99 42	2·18
15	γ Ceti	2 38·1	87 11	3·58	34	α Orionis	5 49·8	82 37	0·93
16	η Eridani	2 51·5	99 18	4·06	35	β Aurigæ	5 52·2	45 4	2·07
17	α Ceti	2 57·1	86 18	2·82	36	γ Cygni	20 18·6	50 4	2·32
18	Algol (β Pers.)	3 1·7	49 26	2·31	37	β Pegasi	22 58·9	62 28	2·61
19	ζ Eridani	3 11·0	99 11	4·88	38	α Pegasi	22 59·8	75 20	2·57

The adopted magnitudes are the means of all values found in the following *Harvard Annals*:—

1. "Harvard Photometry" (vol. xiv.).
2. "Photometric Revision of the D.M." (vol. xxiv.).
3. "Miscellaneous Photometric Measurements" (vol. xxiv.).
4. "Photometric Revision of the Harvard Photometry" (vol. xliv.).
5. "A Photometric D.M." (vol. xlv.).

TABLE II.

Observations of the Magnitude of o Ceti (Mira).

Day and Hour. 1906.	h	Obsr.	Estimations.	Resulting Magnitude.
Dec. 14	7	R	·2 f 29; ·1 b 12; ·4 b 17; =3	2·19
14	9¼	W	much f 24, 30, 31, 32, 35; much b 10	...
21	8½	W	=18; =11; ·5 f 12; ·33 b 17; ·67 f 35; ·5 f 30, 31, 32; 1·5 b 10; much f 24	2·50
26	6¼	R	·2 f 29; ·2 f 12; ·2 f 3; ·7 b 17; ·7 b 9	2·18

TABLE II.—*continued.*

Observations of the Magnitude of o Ceti (Mira).

Day and Hour.		Obsr.	Estimations.	Resulting Magnitude.
1906. Dec. 26	h 8½	W	·5 f 12 ; ·25 b 30 ; ·25 f 31 ; ·25 f 32 ; =11 ; much f 24, 27, 34	2·25
27	8½	W	·2 f 12 ; ·5 b 17 ; =31, 32 ; ·3 b 30 ; ·2 b 11	2·06
27	8½	R	=12 ; ·2 f 1 ; ·3 b 38 ; ·7 b 37 ; ·7 b 17 ; =38 (2nd obs.)	2·24
1907. Jan. 2	6½	R	·2 f 1 ; ·2 f 12 ; ·6 b 9 ; ·4 b 17 ; ·4 f 29	2·30
3	6	R	·3 b 17 ; ·4 b 38 ; ·1 f 37 ; ·6 b 9 ; ·3 f 1 ; ·3 f 12 ; ·6 b 2	2·40
3	6¼	W	·3 f 1 ; =6, 11, 12, 36 ; ·4 b 17 ; ·3 f 20 ; ·2 f 35	2·31
6	8¼	R	·3 b 17 ; ·3 f 12 ; ·6 b 9 ; ·3 f 6	2·46
6	8⅓	W	·5 f 28 ; ·4 f 12 ; ·3 b 9 ; ·3 f 20 ; ·25 b 17 ; ·3 f 36, 35 ; ·2 f 29, 30 ; ·1 b 4 ; =33 ; ½(1+2)	2·39
11	6	R	·5 b 9 ; ·5 f 12 ; ·1 b 17	2·56
11	7	W	·4 f 12 ; ·2 b 9 ; ·3 f 6 ; ·2 b 17 ; ·2 f 32 ; ·2 b 25	2·53
17	8	W	=17 ; ½(9+12)	2·59
17	8½	R	·25 b 9 ; ·3 f 17 ; ·5 b 2 ; ·4 f 33 ; ·4 b 26 ; ·5 b 23	2·63
18	7	R	·15 f 17 ; ·2 b 9 ; ·3 b 2 ; ·4 b 26 ; ·2 f 30	2·64
22	7	R	·4 f 17 ; ·3 b 23 ; ·2 b 26 ; ·4 f 33 ; ·8 b 5 ; ·9 b 7 ; 1·0 b 15 ; ·4 f 30 ; ·15 b 2	2·82
23	6½	R	·2 b 26 ; ·4 f 33 ; ·4 f 17	2·82
29	7	R	=26 ; ·5 f 17 ; ·45 b 15 ; ·2 f 9 ; ·1 f 23 ; ·7 f 33 ; ·1 f 2 ; ·8 b 10	3·08
31	6¾	R	·2 f 23 ; ·2 b 15 ; ·6 f 17 ; ·3 b 10 ; =26 ; ·45 f 2 ; ·55 f 9 ; ·3 b 22 ; ·4 b 21 ; ·4 b 14	3·38
Feb. 2	6¾	R	·7 f 17 ; ·25 b 15 ; ·5 b 14, 10 ; ·4 b 16, 21 ; ·4 f 23, 2 ; ·15 b 22 ; ·5 f 9 ; ·2 f 26 ; ·3 b 8	3·42
2	7	W	1·0 f 17 ; ½(14+17)	3·55
10	8	R	=16, 21 ; ·2 b 14 ; ·1 b 10	3·88
13	7	R	·1 f 14 ; ·2 f 10 ; ·2 b 13 ; ·6 b 19 ; ·2 f 16, 21	4·16
16	7	R	much f 14 ; estimated mag.	4·50

Observers' Remarks.

1906. Dec. 14. Half weight to comparison with No. 3. Colour of Mira a mean tint between the pale yellow of Capella and the deep orange of Aldebaran. (R.)

(With small telescope, aperture 1¾-inch, the image of the variable showed distinct carmine fringes, which were not visible in the comparison stars, with two exceptions, α Ceti and Aldebaran, where the colour and fringes were much fainter. [W.])

Dec. 21, 27. The dull yellow colour of the variable and some

of the comparison stars is obvious to the naked eye, and comparison with brilliant white stars, such as those of the belt of Orion, is difficult. (W.)

Dec. 26. Observations rather difficult owing to the proximity of the Moon. Slight haze at times. Half weight to comparison with No. 3. (R.)

Dec. 27. Mira similar in colour to α Arietis, yellow. (R.) Half weight to comparison with No. 17. (W.)

1907. Jan. 2. Colour of ο Ceti similar to that of α Arietis. (R.)

Jan. 3. α Ceti is reddish; ο Ceti is yellow, closely resembling α Arietis in colour. (R.)

Jan. 6. Colour of ο Ceti same as α Arietis, but α Ceti and β Androm. are orange-red. (R.) The yellow colour of Mira is still manifest to the naked eye. (W.)

Jan. 11. α Ceti reddish; α Arietis and ο Ceti yellow. (R.)

(W. and R. examined Mira Ceti with the Barclay Equatorial, aperture 10-inch, power 88. Both observers noted that the image of Mira showed markedly red spiculæ around the margin, but this margin was not so broad nor so deep a red as the same observers saw around Nova Persei in 1901. Two other coloured stars were examined for comparison. α Ceti exhibited but little red in its fringe, whilst a light orange or canary colour prevailed over the main image. Aldebaran showed reddish orange with a narrow margin of red, but this of a light tint only. The image of Mira Ceti was therefore quite distinct in appearance from these stars when viewed with the same optical means.

Observer R. notes, in addition, that "with the naked eye Mira is not so deep an orange as α Ceti, but with the Barclay telescope it is decidedly deeper than α and not quite so bright.")

Jan. 17. Sky itself transparently clear, but a ground fog prevailed, diminishing from the horizon towards Cetus, but even there it obscured all stars of the fifth magnitude. (W.)

The fog seemed to render stars of an orange colour more conspicuous to-night; α Ceti was, probably from this cause, relatively brighter than usual. The observations were made from the summit of the Observatory tower, 100 feet above the ground. (R.)

Jan. 18. Foggy; estimations difficult; but probably the mean is reliable. The observations were made from the summit of the Observatory tower. (R.)

Jan. 22. Bright Moon effectually screened during the observations. (R.)

Jan. 23. A brief comparison only. (R.)

Jan. 29. Observed in intervals of cloud and in bright moonlight. Estimations rather difficult, but mean is reliable. (R.)

Jan. 31. Sky hazy; observation difficult. (R.)

Feb. 16. Seen with difficulty in moonlight and through low haze. (R.)

Radcliffe Observatory, Oxford:
1907 *April* 9.

Early and Late Perseids. By W. F. Denning.

The following is a list of the apparent paths of probable and possible Perseids observed at Bristol during the 16 nights July 7 to 22 and 9 nights August 17 to 25 inclusive in the thirty years from 1876 to the present time. A proportion of these meteors were undoubtedly not true Perseids, but belonged to the other showers existing in Cassiopeia, Andromeda, Pisces, Aries, Camelopardus, Taurus, Auriga, etc., at the same epoch. There are a great number of such radiants visible—certainly more than a hundred—and it is very difficult to attribute correct positions for some of the individual meteors which have appeared at this time of the year.

The materials here furnished may afford some help in determining the dates of beginning and ending of the Perseid shower. These are very doubtful at present, but I believe that the weight of evidence favours the conclusion that true Perseids may be occasionally recognised after the first week in July. The display is very feeble at that time, and may not supply one meteor during a watch of several hours by a single observer. But on July 19 the stream becomes well pronounced, and its radiant capable of being definitely ascertained, though not in every year. The date just mentioned is in fact the earliest one on which I can confidently say that Perseid meteors are sometimes visible in sufficient numbers to enable a good radiant to be obtained by an individual observer.

In the last column of the list I have ascribed the radiants as they appear in my MS. book of observations. These positions are not correct in all cases. But I have reprojected the apparent paths, and have affixed the letter P to distinguish those which may be regarded as Perseids.

If other observers will supply similar data it will be possible to deduce the place of the radiant on every night from July 7 to 22. But it cannot be safely said that the shower begins so early as July 7; it will be very difficult to learn when the earth really encounters the first shots from this widely-distended stream.

Further materials for the above period, and for the concluding stages from August 17 to 25, will throw an interesting light on the actual duration of the shower, and on the positions of the radiant near the limiting dates of its visibility.

We might soon determine these features but for the fact that there are many other secondary displays yielding similar meteors and radiating from the same region of the sky. It is often impossible to say whether meteors are Perseids or not, though their flight-directions may conform with the position of the great August shower. At the earlier stages of the display there are well-defined minor radiants at about

23°+43°	33°+52°	50°+31°
23+57	43+22	54+71
29+36	44+57	60+58
33+18	47+43	61+48

—and many others, varying in strength from year to year.

With regard to the concluding phases of this system, I have recorded meteors from the right position up to August 25, but there are doubts as to whether the shower is really prolonged to that date. It is certainly continued to August 20, but on subsequent nights the evidence gleaned is not sufficiently ample to enable one to absolutely affirm its visible continuance. From a discussion of my combined observations in various years, I have deduced the following radiants of Perseids :—

Aug. 16	53+58	21 ↓ *s*	Aug. 20	57+59	10 ↓ *s*
,, 17	54+60	5 ,,	,, 21	64+59	12 ,,
,, 18	55+59	11 ,,	,, 22	70+61	7 ,,
,, 19	57+59	5 ,,			

Companion showers (in addition to those above given for the earlier stages) are well pronounced at about—

39+28	70+66	75+15
58+ 9	71+52	77+32
61+36	74+42	87+43
63+22	77+58	106+52

The earliest radiant I have obtained, presumably for the Perseids (though a shower of Cassiopeids may also be involved) is for the period July 7–9, 10°+45½°, ten meteors. Quite possibly the shower continues during fifty nights July 7–August 25.

The late Mr J. Kleiber gave the theoretical place of the Perseid radiant as, on July 8, 9°+46°, and August 16, 54°+59°—(*Monthly Notices*, lii. p. 351).

Observed Paths of Early Perseids.

Date.	G.M.T. h m	mag.	From α δ	To α δ	Path.	Notes.	
1877 July 7	13 21	2	349+25	345+20	6½	R K	P
1902 ,,	11 44	3	318+56	283+49	22	R K	P
1885 8	12 1	2	240+61	218+37	27	v R B K	30+8
1888 ,,	11 4	4	47+71	87+73	12	R	345±0
,,	12 7	4	61+59½	81+70	7	R K	
,,	12 36	3	355+22	349¾+10½	11½	R K	P
,,	13 22	2	332+72½	320+75	4½	R B K	P
1902 9	13 11	3	310+20	303½+14½	8	R K	P
1877 11	11 18	2	339+26	316+3	31	v RK	47+43
1885 ,,	11 62	2	278+45	260+32	18	v R	P
1888 ,,	11 36	3	344+58	334+59	5½	S B K	P
,,	11 49	4	344+59½	333+61½	6	S K	P

Observed Paths of Early Perseids—continued.

Date.	G.M.T. h m	mag.	From α δ	To α δ	Path.	Notes.
1904 July 11	14 14	2	18°+36°	20½°+32½°	4°	R K P
1877 12	11 43	3	335+48	313+40	17	R 47+43
,,	11 57	3	336+30	313+11	28	v R P
,,	12 17	3	340+64	308+66	13	v R K 23+43
1885 ,,	11 18	3	281−15	275−21½		S K P
1888 ,,	12 5	3	44+58½	54+60	6	R K P
1885 13	12 9	2	293½+35½	276½+20	21	v R K P
,,	12 49	3	356+41	350+37½	6	R K P
14	12 35	3	324+64	279+62	20	v R B K P
1901 15	13 5	>1	270+53	248½+32	24	R K 23+43, 29+36
1876 16	11 17	1	260+80	225+67	15	R K 23+43, 47+43
,,	11 33	2	310+51	286+35	22	v R K P 47+43
,,	12 19	2	321+42	304+30	17	R B K P
1898 ,,	11 4	3	24+53	26+57	4	S K P
1902 ,,	13 47	5	15½+55	14½+58	3	R K 20+43 P
1898 ,,	12 12	3	348+53	339+53½	5½	R K 23+43
,,	12 42	4	8+66	0+71	6	S B K 23+43
1898 17	12 42	4	25½+54½	27+59	5	R K 23+43
1876 18	10 35	4	16+47	15+40	7	R 27+71
,,	11 20	>1	211+49	206+18	31	R K P
1881 ,,	11 9	1	110+70	144+58½	19	R K P
1887 ,,	11 1	>1	3+34½	0+31	4	S B K P
1900 ,,	10 38	5	314+46½	306+42	8	R K P
1876 *19	10 58	1	346+25	337+12	20	B K P
1887 ,,	11 43	1	358½+38	351+30	10	S B K P
,,	12 25	1	350+62	330½+64	10	S B K P
,,	13 8	4	356+47	347+44	7	R K P
1900 †,,	11 43	1	344+33	329+18	20	R K P
1901 ,,	11 41	1	335+68	289+67¼	12½	v R K 23+43, 29+3
,,	12 58	4	346+35	339½+30	7	R K P
1876 20	11 29	1	341+6	335−8	15	K P
1901 ,,	11 24	4	338+60	302+58½	8	v R K 23+43, 29+3
,,	11 59	2	328+59	292+54¼	20	v R K 23+43, 29+3
1876 21	11 21	2	344+60	315+57	15	R K P
1887 ,,	12 40	3	5+44	359½+39	6½	v R P

* Observed also by J. Lucas, Oxford. Radiant, 22°+54°.
† Observed also by Prof. A. S. Herschel. Radiant, 17+50.

Observed Paths of Early Perseids—continued.

Date.	G.M.T. h	m	mag.	From α	δ	To α	δ	Path.	Notes.	
1901 July 21	11	57	5	53°	+62½°	70°	+65½°	8°	R K	23+43
,,	12	2	5	25	+36	25	+32	4	R K	23+43
,,	13	40	3	311	+52	297½	+45	11	R K	P
,,	13	43	1	349½	+15	341	+1½	16	R K	P
,,	14	0	3	26	+53½	31	+53¾	3	S K	P
1887 22	11	2	4	6	+43	1	+40	5	R K	P
,,	12	21	1	356	+45	332	+29	24½	R B K	P
,,	13	2	5	25	+47	27	+43	4	S	P
,,	13	55	3	331	+77	281	+75	12	R K	P
1900 ,,	11	15	2	13	+55	356	+59¾	10	S K	P

Late Perseids.

Date.	G.M.T. h	m	mag.	From α	δ	To α	δ	Path.	Notes.	
1901 August 17	15	21	2	90°	+43°	98°	+35°	10°	R	P
1885 ,,	10	59	4	346	+59	318	+46	21	R K	P
,,	12	44	4	304½	+46	298½	+39	8	v R K	P
1898 ,,	9	36	3	55½	+69	57½	+74	5	S K	P
1899 ,,	15	8	2	96	+43	103	+36	8½	R K	P
1885 18	13	37	3	335	+37	328	+27½	11	R K	P
1893 ,,	12	25	♀	330	+31	315½	+9	26	R K	P
,,	12	37	1	38	+10	37½	+1	11½	R B K	P
1901 ,,	10	28	3	327	+40	320	+21	23	R K	P
,,	10	54	3	329	+50	313	+34½	17	R K	P
,,	11	44	3	241	+84	236	+72	12	R K	P
,,	12	16	5	4½	+10	0	+2	9	R	P
,,	12	48	5	69	+63½	83	+65½	6	R K	P
,,	12	53	2	37	+64½	25	+66¾	5½	S B K	P
,,	13	4	2	245	+86	236	+71	15	R K	P
1884 19	11	10	3	88	+59	92	+59½	2½	v R	77+58
,,	11	27	3	39	+68	23	+70	6	K 4°	P
,,	12	21	4	341	+78½	308	+73¾	9	B K	P
1901 ,,	11	48	4	353½	+16	348	+9	9	R K	P
,,	12	29	3	22½	+58½	14	+57	5	S K	P
1885 20	11	41	3	28	+46	24	+42	5	R K	P
,,	13	55	3	352½	+15	347½	+6	10	R K	P

Late Perseids—continued.

Date.	G.M.T. h	m	mag.	From α	δ	To α	δ	Path.	Notes.
1887 August 20	10	1	5	356°	+67°	297°	+14°	64°	S K 77+58
1901 ,,	10	16	>1	349+43		327½+19½		29	R B K P
,,	10	35	3	79+56½		92+53		8	S P
,,	11	2	4	28½+22½		25½+15½		8	R P
,,	11	41	3	72+67½		82+71		5	S BK P
,,	11	45	5	357½+32½		350+22		11	R K P
,,	13	58	4	308½+50		299½+40		11½	R P
1884 21	11	25	3	16+56		3+54		8	R K 71+52
,,	11	40	2	42+81		309+84		11	R BK 71+52
1887 ,,	10	37	5	11+50½		0+44		10	R K 77+58
,,	12	58	4	27+49		18+45½		7	R K 71+52
1901 ,,	10	0	4	312½+51½		303+41		12	R K P
,,	12	20	2	14+45		359+33½		15½	R K P
,,	12	48	2	22½+17		16+3½		15	R K P
,,	13	8	4	17+15		11½+3		13	R K P
,,	15	11	5	66+36		66+31		5	R K P
1903 ,,	13	4	4	30+37½		27+33		5½	R K P
1884 22	12	21	4	72+48¾		69+43½		5½	not R K 77+58
1900 ,,	10	20½	3	287+78½		264+67		13	v R K P
,,	11	39	4	17½+28		11+17		13	R K P
,,	11	48	3	11+44¼		3+37½		8	R K P
1901 ,,	10	52	4	334+62		319+52½		12	R K P
,,	12	5	3	11½+10		7+2		9	R K P
1906 ,,	11	8	>1	18+38		7½+28		13	S K P
,,	11	9	5	345+33		336+22		15	v R K P
1884 23	11	59	1	345+19		335½+2		19½	v R K 40+59
1887 ,,	12	27	3	93+62½		109+60¼		8	R K 72+62
1900 ,,	11	3	4	16+22		12+16		7	R 77+58
1903 ,,	11	0	4	25+27		22+21		7	not R K 70+65
1887 24	11	10	3	66+70		6+81½		17½	R K 77+58
1900 ,,	9	26	2	25+51		10+42		13	v R K 77+58
1879 25	11	0	3	243+30		243¾+16		14	R K 62+35
1903* ,,	9	35	3	27+29		20+19		12	R K 79+59

Abbreviations—R, rapid; S, slow; B, bright; K, streak; v, very; P, Perseid.

* Also observed by Mr C. L. Brook at Meltham, Huddersfield.

The Electric Arrangements of an Observatory.

By W. Ernest Cooke, M.A.

A system of electric connections for the numerous instruments has been evolved at the Perth observatory of so simple and convenient a character that a description may be interesting and perhaps useful, especially to those who contemplate establishing a new observatory.

There is only one sidereal clock and one battery used for all purposes, such as chronographs, clock dials, clock control, equatorial control, chronograph control, transit keys, and small electric lights.

The battery consists of two large accumulator cells having an E.M.F. of 4 volts. There are in reality two of these used alternately, so that one may be always available. It is found advisable to change the battery at 9 a.m. daily, and to recharge the one not in use. The charging is performed very simply by means of a motor transformer, worked from the ordinary electric light supply current.

By means of a three-way switch either battery can be turned on or off, or connected with the charging transformer, instantly.

From the battery a pair of heavy mains are run round the buildings and connected to a pair of terminals in each room, wherein any instruments which require an electric current are located. These terminals are labelled A and B.

From the contacts of the sidereal clock a current passes through a relay, and the contacts made by the armature of this relay practically indicate the adopted seconds of the clock. It has been considered advisable, in the case of the clock contacts only, to use a separate battery, and two cells of the gravity type are found to give sufficient current. But even in this case a switch has been provided for substituting the current from A B if the gravity battery is out of order.

One of the terminals of the secondary contact of the relay is connected to A, and the other to a third main wire which runs round all the buildings and is labelled C. The function of the clock relay is thus to make a momentary connection between A and C at every beat of the sidereal clock.

We have thus, in every instrument room, the three terminals A, B, and C, and from these all necessary connections can be made. Thus, for a chronograph or clock dial, wires must be run direct from C B, and this gives an electric impulse every second, since C and A are joined once a second by the sidereal clock relay. For an instrument which works by means of a key, or any form of hand control, wires must be run from A B. If a chronograph be of a

single pen type, *i.e.* if both the clock beats and the key taps are to be registered by means of the same electro-magnet, it will be necessary to introduce a relay; for otherwise the key will connect A and C, and will send an extra impulse along the clock circuit at every tap. In this case the primary terminals of the relay are connected with C B direct, and the secondary with A B through the chronograph. The secondary of the relay is thus placed in parallel with the key.

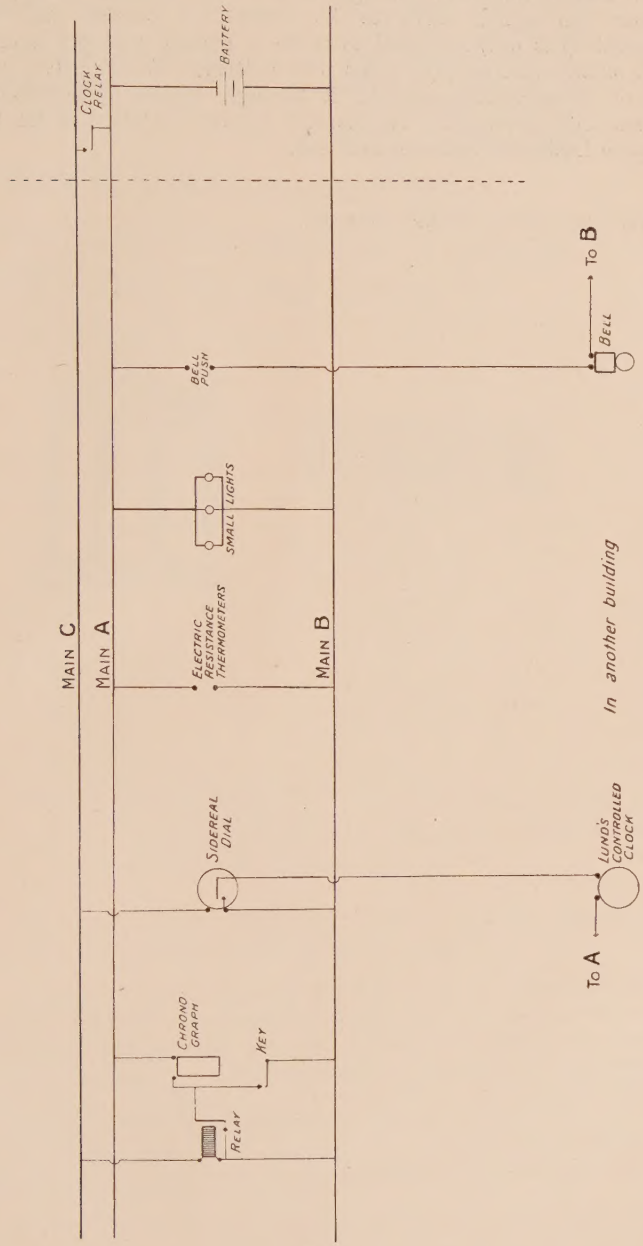
It may be mentioned that one of these relays is used as a key for transmitting clock beats in determining differences of longitude.

The system is especially convenient when it is desired to control an instrument in one building by means of a key or instrument in another building. For instance, there is a clock dial in the transit room, worked from the sidereal clock by simply connecting its terminals with B C. This dial contains a pair of contacts which are joined for a couple of seconds at each sidereal hour. These are required to control, by means of Lund's clips, a clock keeping sidereal time in another building. The connections are very simple. One of the primary contacts of the dial is already connected with B. It is now also joined to one of the secondary contacts. The other secondary is connected to a single line wire which leads to one of the terminals of Lund's control, and the other terminal is joined to the nearest A. The arrangement is shown in the diagram. Even the line wire might be dispensed with and an earth used if this instrument only were concerned, but it will be found advisable to keep away from earth connections entirely.

The accompanying diagram gives a rough sketch of the manner in which the system is used, but conveys no idea of the number of instruments which may be connected. At the Perth Observatory the following are all worked by the one battery:—

Two sidereal dials.
Three sidereal chronographs.
One mean time chronograph.
Three transit keys.
One complete longitude system for short distances.
One Lund's controlled clock.
One mean time multiple relay.
One electric recorder (for platinum thermometers).
Two complete Grubb's controls (Astrograph and Chronograph), using the sidereal clock instead of an auxiliary pendulum.
One control for mean time clock.
Several small lamps on instruments.
Two sounders for beating seconds in dark rooms.
Four call bells.
Two alarm bells on Astrograph.

All electric connections are, in fact, worked from the A B C terminals except the hourly current from the mean time clock.


MAIN C
MAIN A
MAIN B
CLOCK RELAY
BATTERY
BELL PUSH
SMALL LIGHTS
ELECTRIC RESISTANCE THERMOMETERS
SIDEREAL DIAL
CHRONO GRAPH
KEY
RELAY
TO B
BELL
In another building
LUND'S CONTROLLED CLOCK
TO A

Even that is connected through A B to the multiple relay, but thence the signals have to be distributed outside, and the observatory is only supposed to make a contact once per hour,—each outside user supplying his own battery. We ourselves take one of these circuits and run it through various meteorological instruments, fire a gun, and control a public clock, but for this purpose Leclanché batteries are used.

Perth Observatory, Western Australia.